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Introduction to Quantum Computing

Lecture4: Postulate IV – Measurement

Raul Garcia-Patron Sanchez

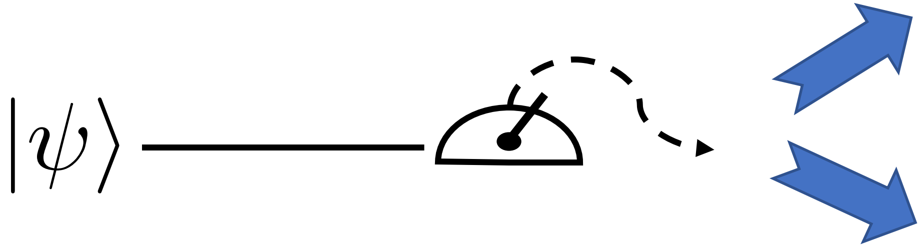
Basis: Computational basis $\mathcal{H} \equiv \mathbb{C}^d$

Computational basis $\quad \mathcal{H}_{\mathcal{Q}} = \text{Span}\{|0\rangle, |1\rangle\}$

- $\forall |\psi\rangle, \exists \psi_0 \text{ and } \psi_1 : |\psi\rangle = \psi_0 |0\rangle + \psi_1 |1\rangle$
- $\langle 0|1\rangle = 0$ (Orthogonal basis)
- $||\,|0\rangle|| = ||\,|1\rangle|| = 1$ (Normalized basis)

This ensure logical 0 and 1 is an orthonormal basis of a Hilbert space of dim 2.

Computational basis measurement



$$P(0) = |\langle 0|\psi\rangle|^2 = |\psi_0|^2$$

$$P(1) = |\langle 1|\psi\rangle|^2 = |\psi_1|^2$$

The amplitudes of the quantum state on the logical basis (0 and 1) are associated with the outcome probabilities of the computational basis measurement (logical 0 or 1).

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The effect of measurement

Random coins vs quantum coins

$$R = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$$

- Classical coin flip

$$\bar{v}_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ — } \boxed{R} \text{ — } \bar{p}_{out} = R\bar{v}_0 = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

- Quantum random number generator circuit

$$|0\rangle \text{ — } \boxed{H} \text{ — (measurement)}$$

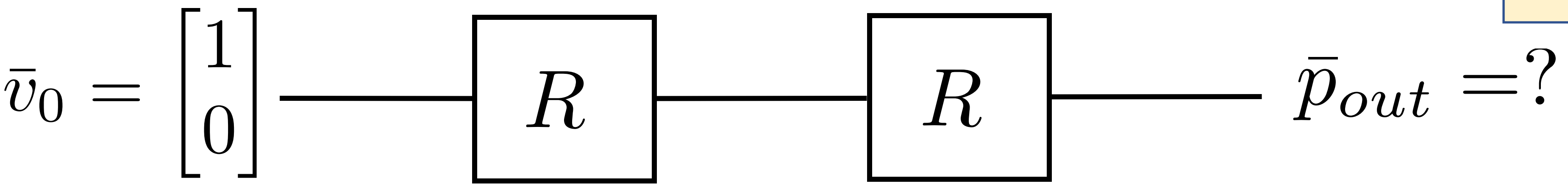
$$P(0) = |\psi_0|^2 = 1/2$$

$$P(1) = |\psi_1|^2 = 1/2$$

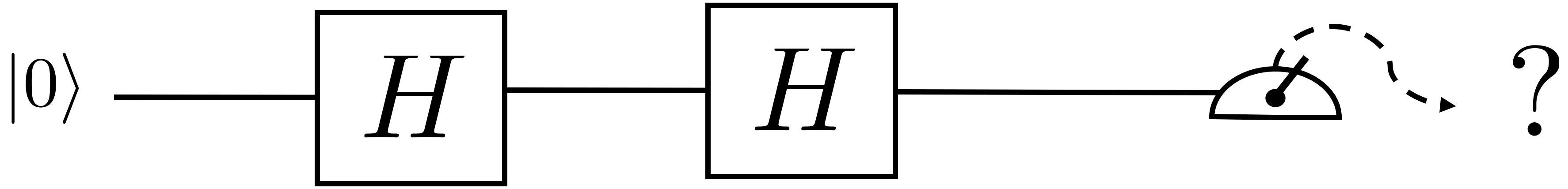
Random coins vs quantum coins: concatenation

$$R = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$$

- Classical coin flip



- Quantum random number generator circuit

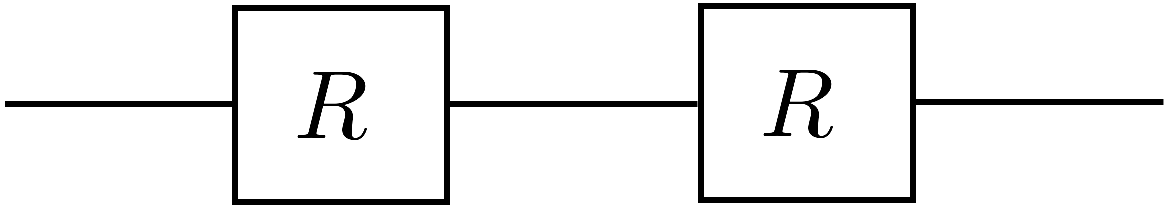


Random coins vs quantum coins: concatenation

$$R = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$$

- Two classical coin flips

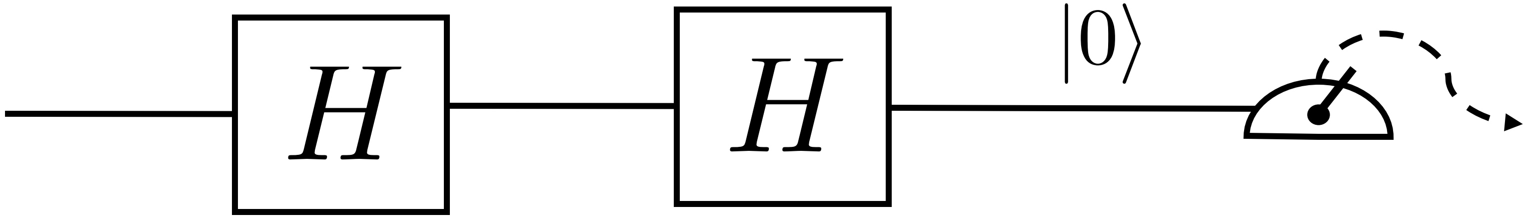
$$\bar{v}_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$



$$\bar{p}_{out} = R^2 \bar{v}_0 = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

- Two "quantum coin flips"

$|0\rangle$

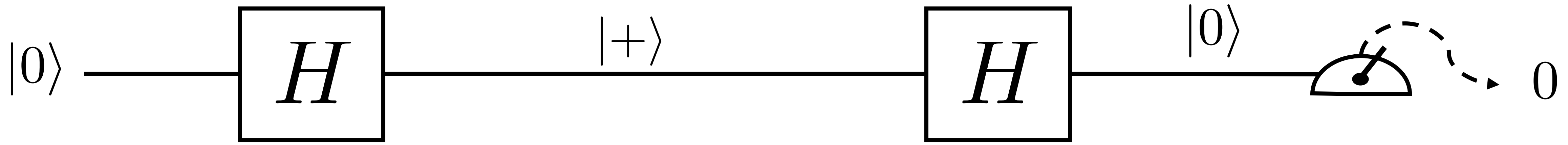


0

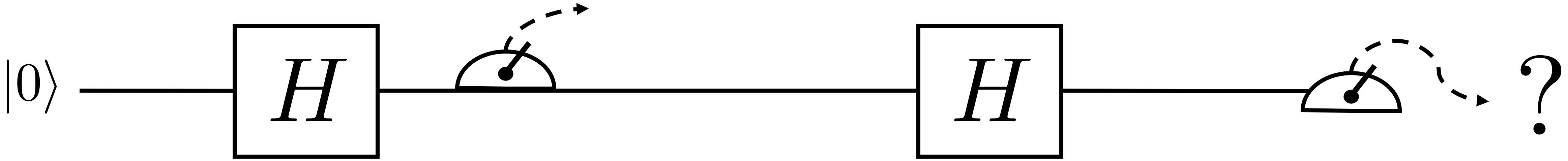
$$H^2 = I$$

To measure or not to measure....

- Two "quantum coin flips"



- Two "quantum coin flips" with intermediate measurement

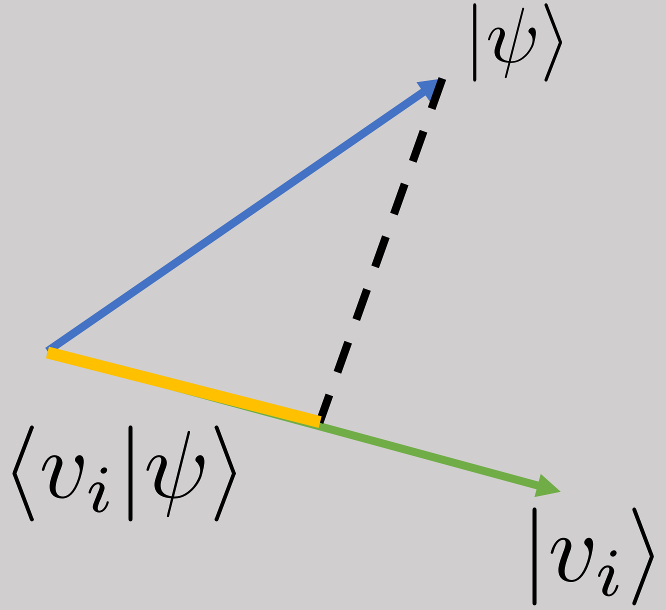


Measurement of orthonormal basis

Any orthonormal basis $\{|v_i\rangle\}$ that span $\mathcal{H}$ has and associated measurement

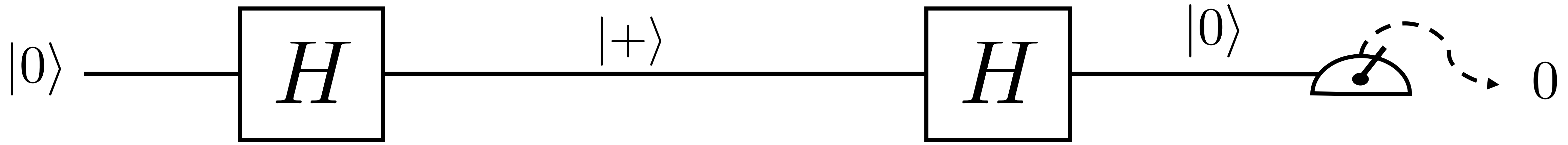
Probability of outcome i reads: $P(i) = |\langle v_i|\psi\rangle|^2$

The quantum state is updated to $|v_i\rangle$

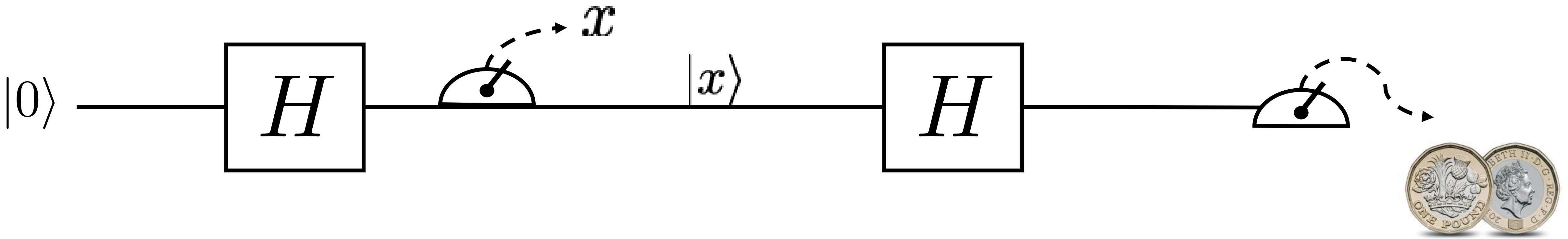


To measure or not to measure....

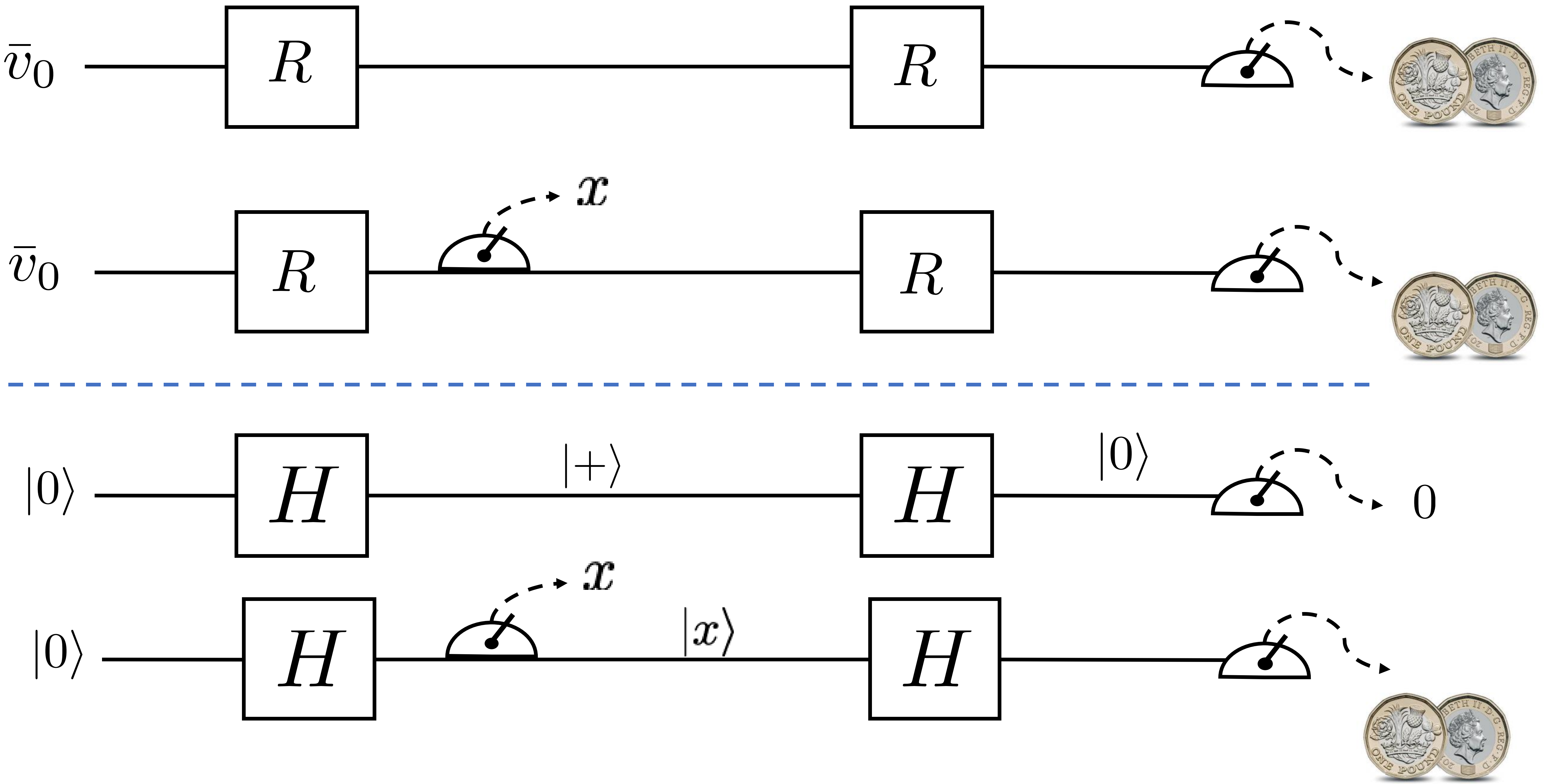
- Two "quantum coin flips"



- Two "quantum coin flips" with intermediate measurement



In quantum world observing the system can change its dynamics!!



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Arbitrary basis qubit measurement

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Basis: General case $\mathcal{H} \equiv \mathbb{C}^d$

- Spanning set of $\mathcal{H}$: set of vectors $|v_1\rangle, |v_2\rangle, \ldots, |v_n\rangle$

$$\forall |\psi\rangle \in \mathcal{H} : |\psi\rangle = \sum_{i=1}^{n} \psi_{v_i} |v_i\rangle \qquad \text{where amplitudes are given by: } \psi_{v_i} = \langle \psi | v_i \rangle$$

- Linearly independent: $\nexists\, a_1, a_2, \ldots, a_n \neq 0$ complex numbers

$$a_1|v_1\rangle + \ldots + a_n|v_n\rangle = 0$$

- Basis: $\text{Span}\{|v_i\rangle\} = \mathcal{H} \Leftrightarrow n = d$
  $+$ Lin. ind.

- Orthonormal: $\forall i, j \in \{1, \ldots, d\}, \langle v_i | v_j \rangle = \delta_{i,j}$

- Has an associated measurement

Example

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

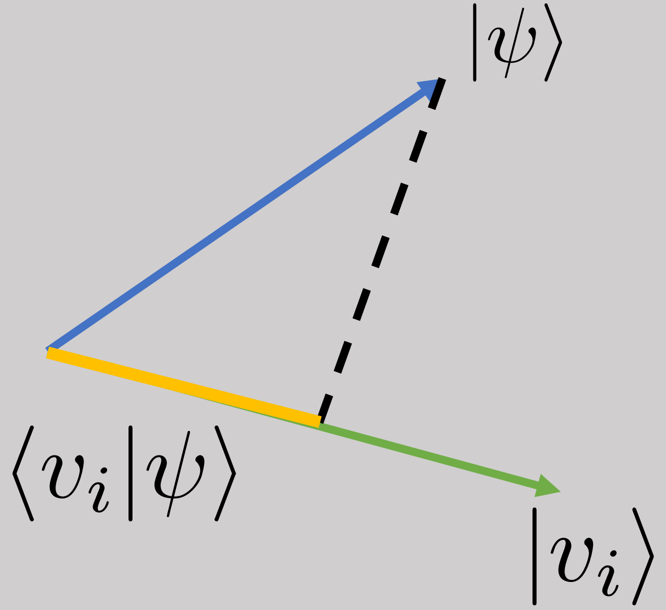
$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Measurement of orthonormal basis

Any orthonormal basis $\{|v_i\rangle\}$ that span $\mathcal{H}$ has and associated measurement

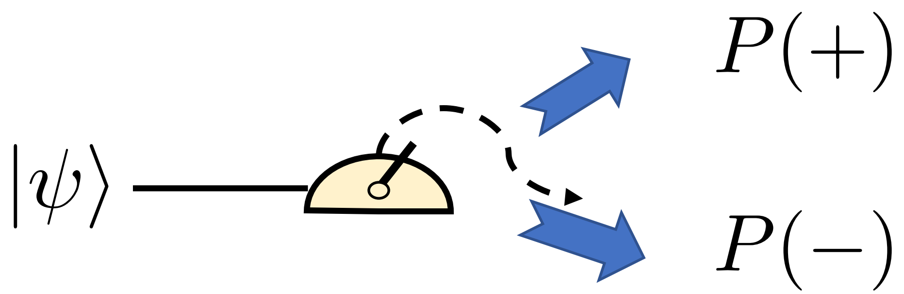
Probability of outcome i reads: $P(i) = |\langle v_i|\psi\rangle|^2$

The quantum state is updated to $|v_i\rangle$



Example:+/- basis

- $|\psi\rangle = \psi_0|0\rangle + \psi_1|1\rangle$



$$\mathcal{H}_{\mathcal{Q}} = \mathrm{Span}\{|+\rangle, |-\rangle\}$$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

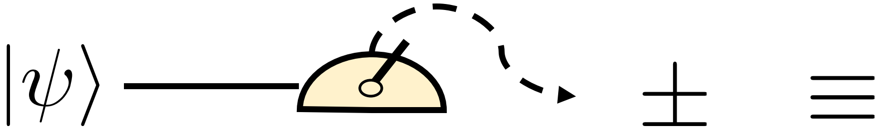
$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

- $P(+) = |\langle +|\psi\rangle|^2 = |\frac{1}{\sqrt{2}}(\langle 1| + \langle 0|)(\psi_0|0\rangle + \psi_1|1\rangle)|^2 = |\psi_0 + \psi_1|^2/2$

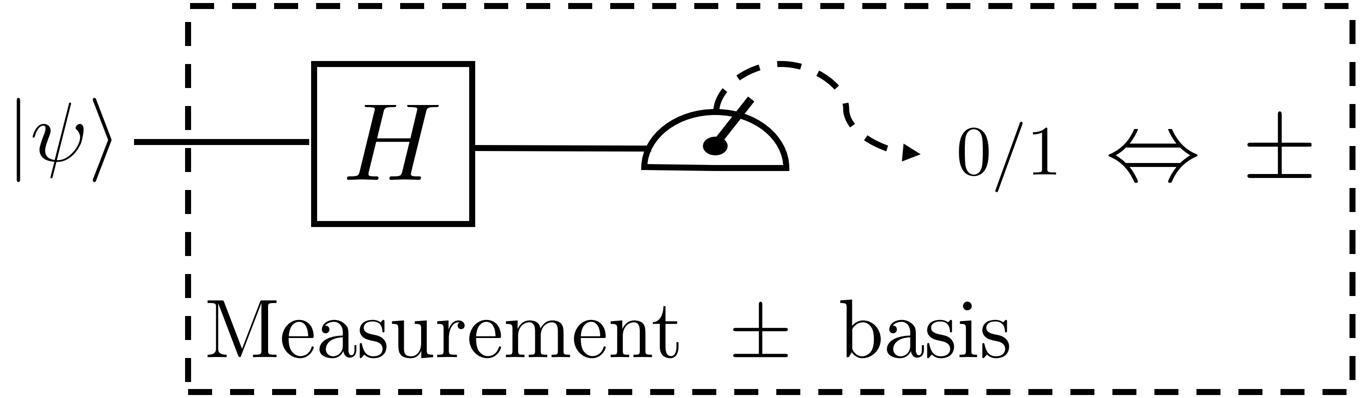
- $P(-) = |\langle -|\psi\rangle|^2 = |\psi_0 - \psi_1|^2/2$

Arbitrary basis measurement

- Measurement basis $\{|\pm\rangle\}$:



$$P(\pm) = |\langle\pm|\psi\rangle|^2$$

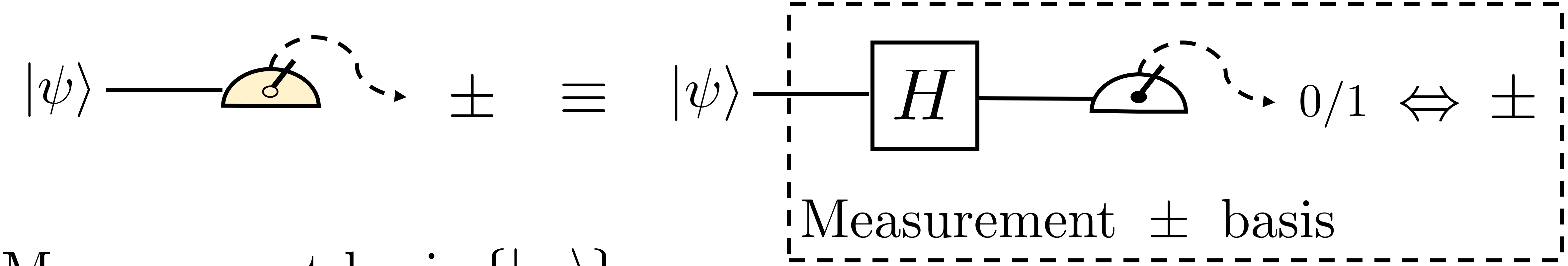


$$|+\rangle = H|0\rangle$$
$$|-\rangle = H|1\rangle$$
$$(A|\psi\rangle)^\dagger = \langle\psi|A^\dagger$$
$$H^\dagger = H$$

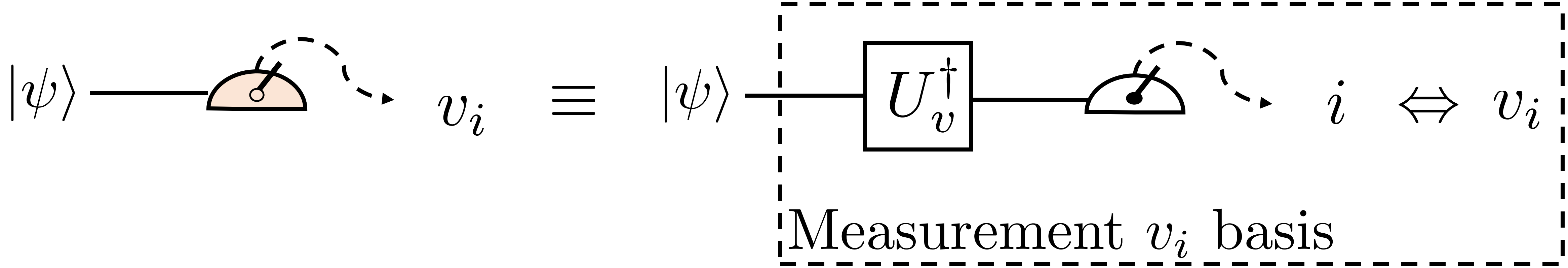
$$\langle+|\psi\rangle = \langle 0|H|\psi\rangle$$
$$\langle-|\psi\rangle = \langle 1|H|\psi\rangle$$

Arbitrary basis measurement

- Measurement basis $\{|\pm\rangle\}$:



- Measurement basis $\{|v_i\rangle\}$:



$\forall$ basis $\{|v_i\rangle\}, \exists U_v$ s.t. $|v_i\rangle = U_v|i\rangle$ $\quad\quad$ $\langle v_i|\psi\rangle = \langle i|U_v^\dagger|\psi\rangle$

References

Reading references

1. NC 2.2.3 and 2.2.5

NC ≣ Michael Nielsen and Isaac Chuang, Quantum Computing and Quantum Information
Cambridge University Press (2010)

Lecture 10 we will revisit the measurement in terms of projectors

There is even a more general concept of measurement in N&C called POVM (2.2.6) that we will not cover in this course.

Even more general is the concept of quantum instrument.

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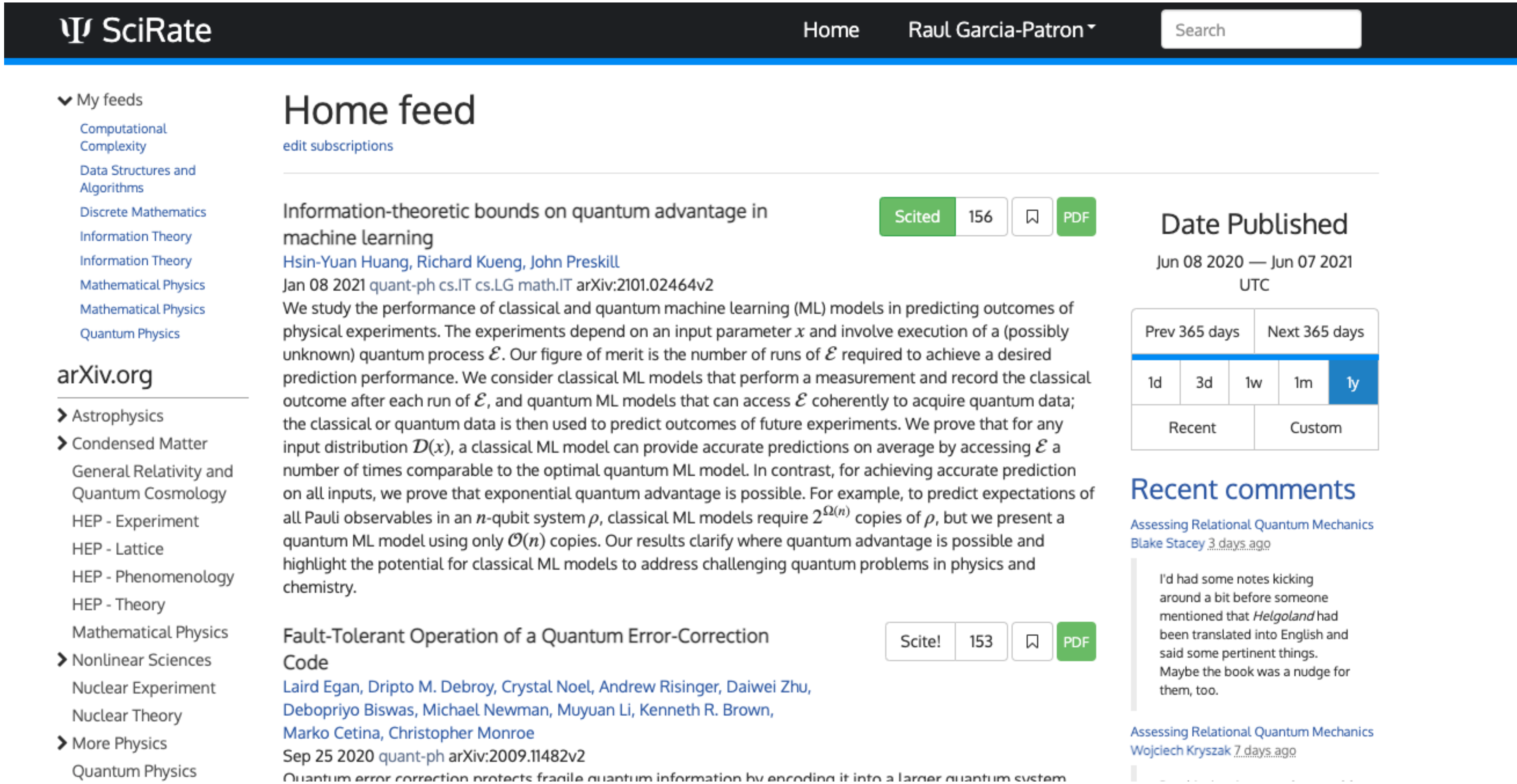

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