

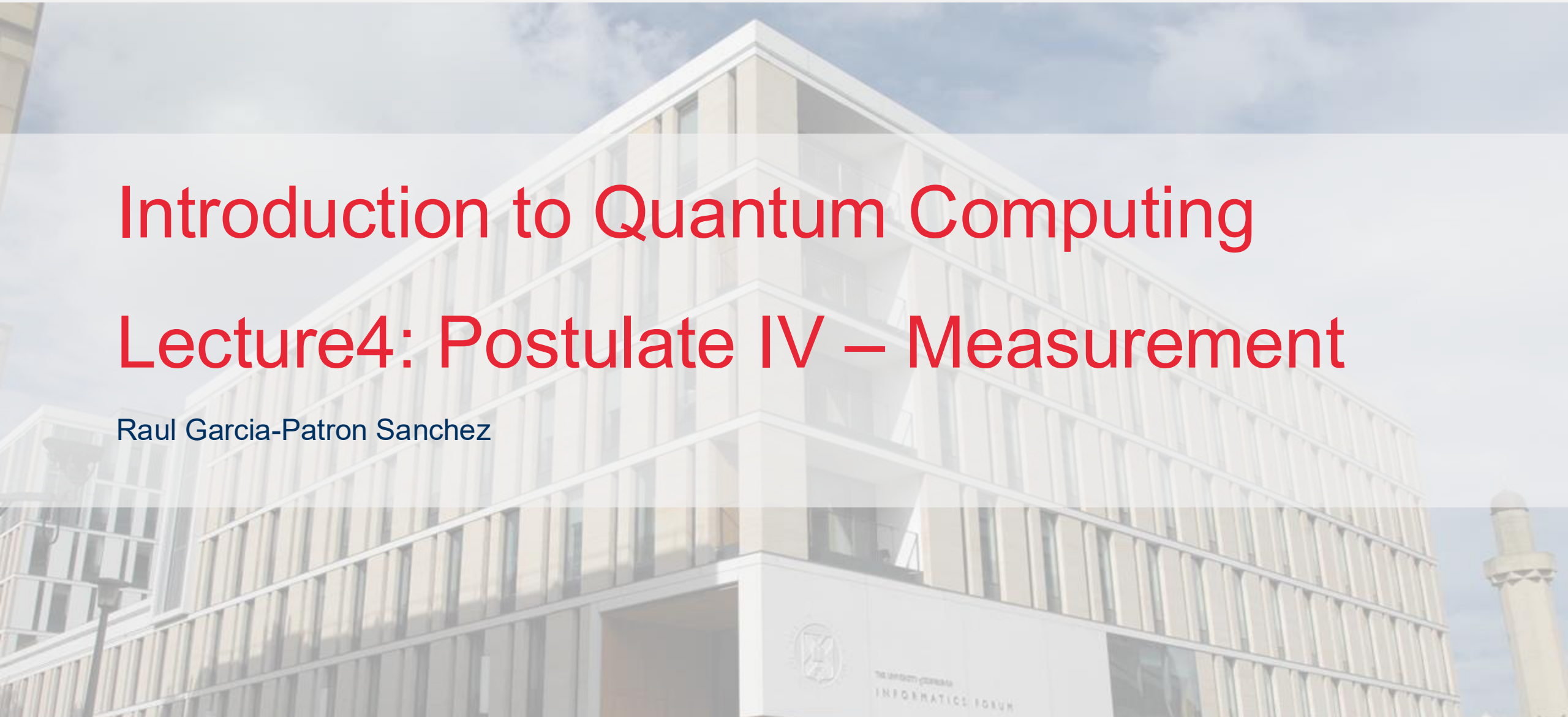


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Introduction to Quantum Computing

Lecture4: Postulate IV – Measurement

Raul Garcia-Patron Sanchez



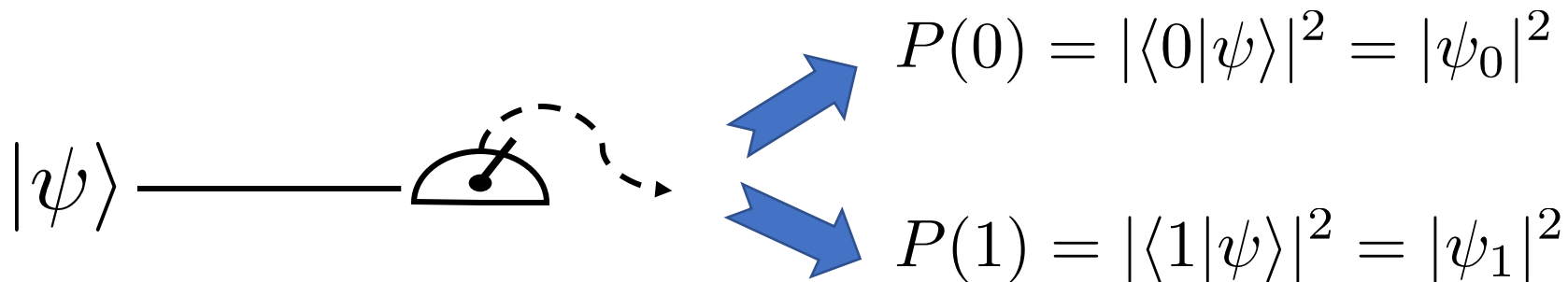
Computational basis

$$\mathcal{H}_{\mathcal{Q}} = \text{Span}\{|0\rangle, |1\rangle\}$$

- $\forall |\psi\rangle, \exists \psi_0 \text{ and } \psi_1 : |\psi\rangle = \psi_0|0\rangle + \psi_1|1\rangle$
- $\langle 0|1\rangle = 0$ (Orthogonal basis)
- $||0\rangle|| = ||1\rangle|| = 1$ (Normalized basis)

This ensure logical 0 and 1 is an orthonormal basis of a Hilbert space of dim 2.

Computational basis measurement



The amplitudes of the quantum state on the logical basis (0 and 1) are associated with the outcome probabilities of the computational basis measurement (logical 0 or 1).



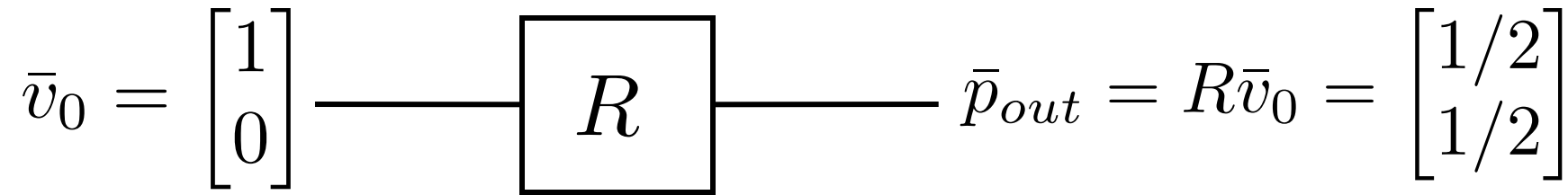
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The effect of measurement



Random coins vs quantum coins

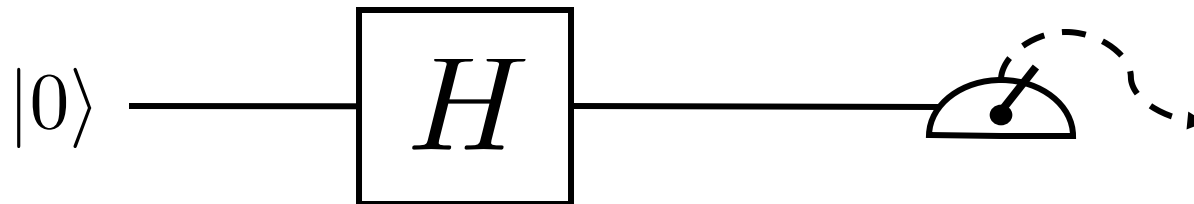
- Classical coin flip



$$R = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$$



- Quantum random number generator circuit



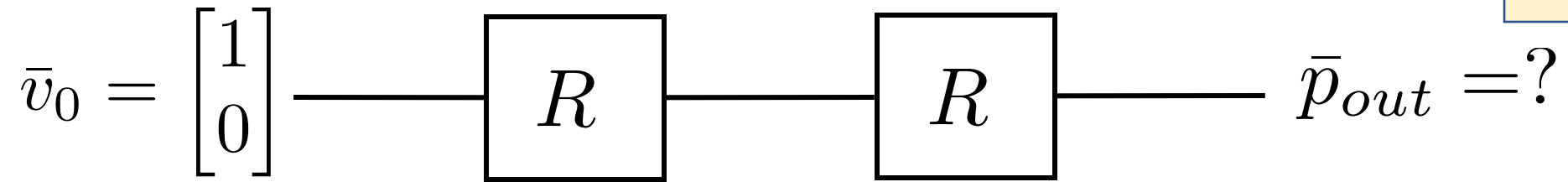
$$P(0) = |\psi_0|^2 = 1/2$$

$$P(1) = |\psi_1|^2 = 1/2$$



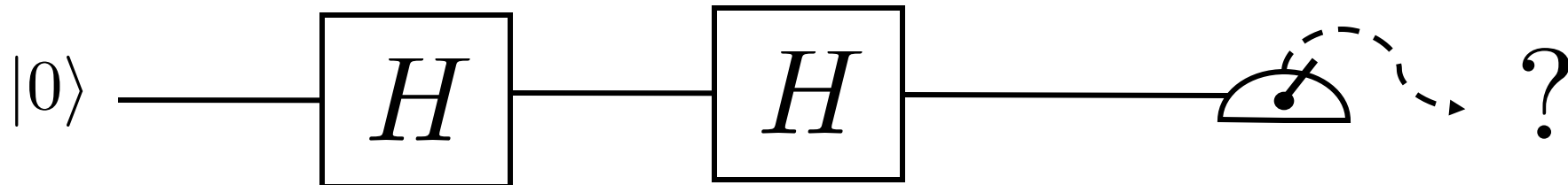
Random coins vs quantum coins: concatenation

- Classical coin flip



$$R = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$$

- Quantum random number generator circuit



Random coins vs quantum coins: concatenation

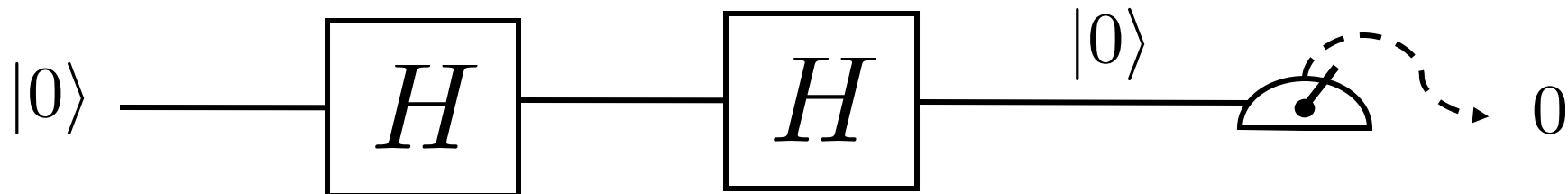
- Two classical coin flips

$$R = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$$

$$\bar{v}_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \longrightarrow \boxed{R} \longrightarrow \boxed{R} \longrightarrow \bar{p}_{out} = R^2 \bar{v}_0 = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$



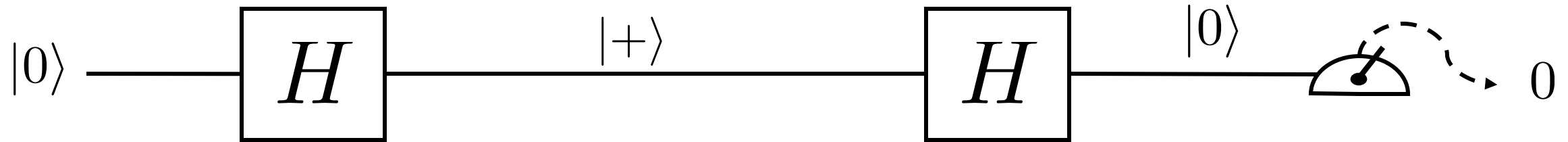
- Two "quantum coin flips"



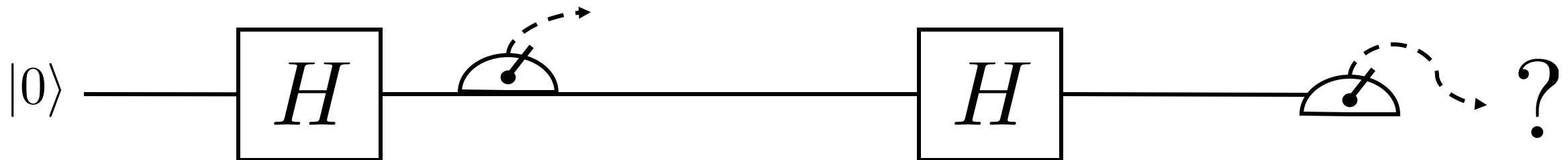
$$H^2 = I$$

To measure or not to measure....

- Two "quantum coin flips"



- Two "quantum coin flips" with intermediate measurement

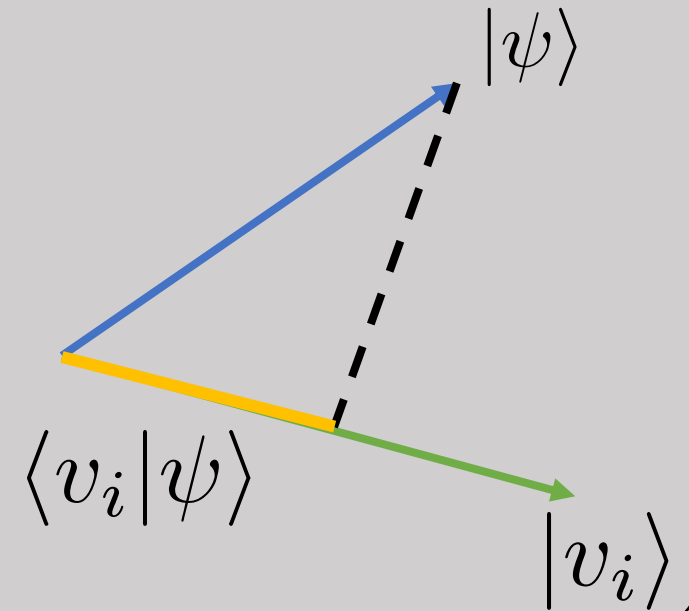


Measurement of orthonormal basis

Any orthonormal basis $\{|v_i\rangle\}$ that span \mathcal{H} has an associated measurement

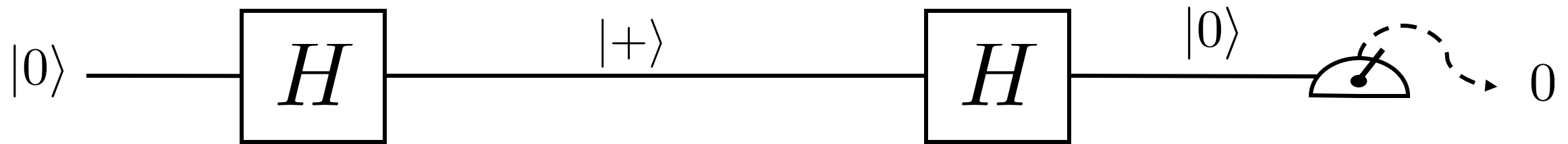
Probability of outcome i reads: $P(i) = |\langle v_i | \psi \rangle|^2$

The quantum state is updated to $|v_i\rangle$

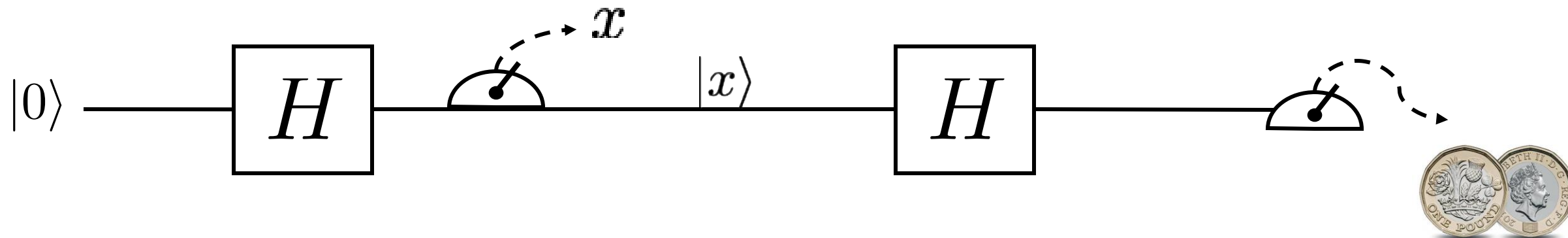


To measure or not to measure....

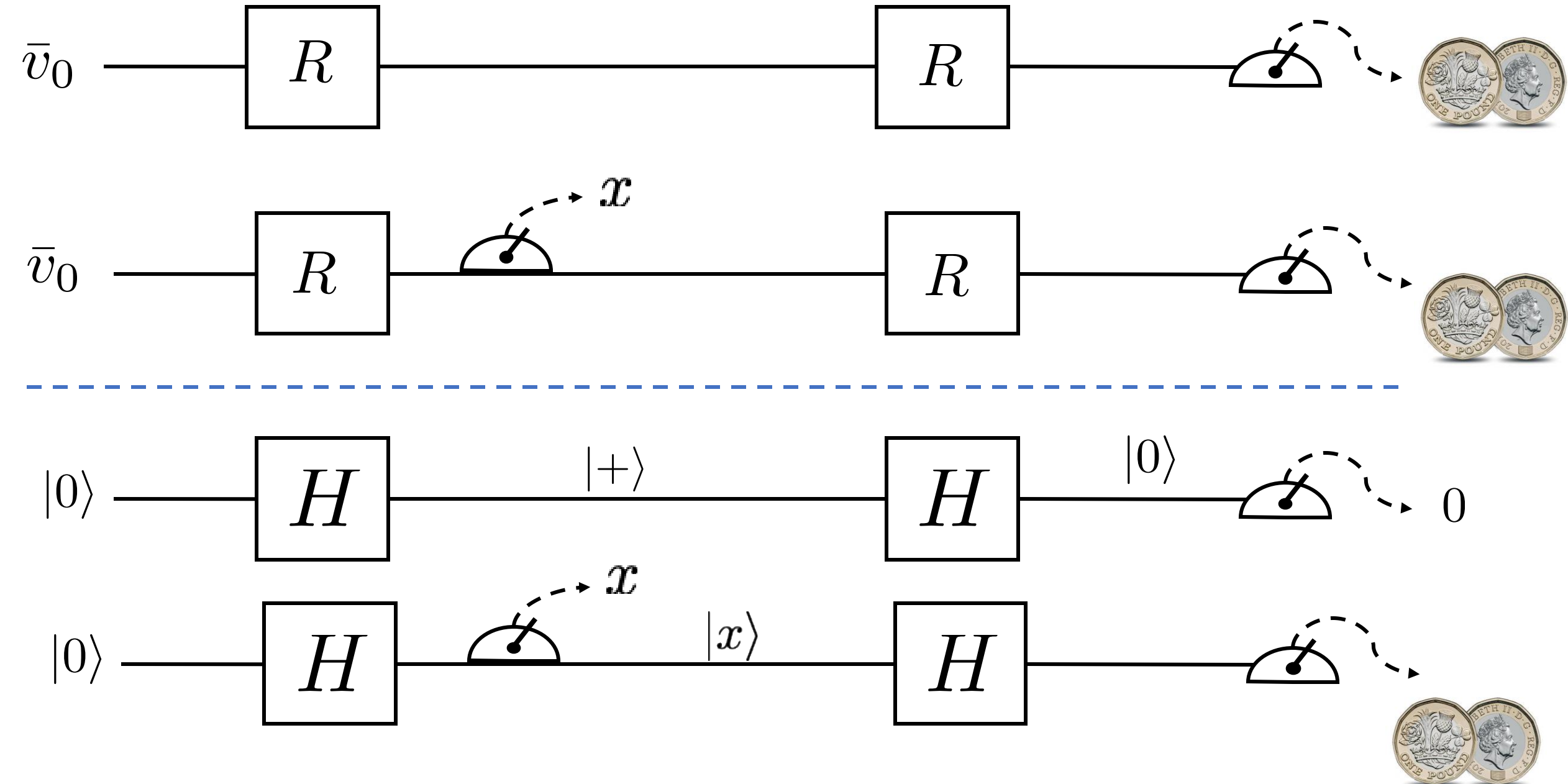
- Two "quantum coin flips"



- Two "quantum coin flips" with intermediate measurement



In quantum world observing the system can change its dynamics!!





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Arbitrary basis qubit measurement

Raul Garcia-Patron Sanchez



- Spanning set of \mathcal{H} : set of vectors $|v_1\rangle, |v_2\rangle, \dots, |v_n\rangle$

$$\forall |\psi\rangle \in \mathcal{H} : |\psi\rangle = \sum_{i=1}^n \psi_{v_i} |v_i\rangle \quad \text{where amplitudes are given by: } \psi_{v_i} = \langle \psi | v_i \rangle$$

- Linearly independent: $\nexists a_1, a_2, \dots, a_n \neq 0$ complex numbers

$$a_1 |v_1\rangle + \dots + a_n |v_n\rangle = 0$$

- Basis: $\text{Span}\{|v_i\rangle\} = \mathcal{H} \Leftrightarrow n = d$
+ Lin. ind.

- Orthonormal: $\forall i, j \in \{1, \dots, d\}, \langle v_i | v_j \rangle = \delta_{i,j}$

- Has an associated measurement

Example

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

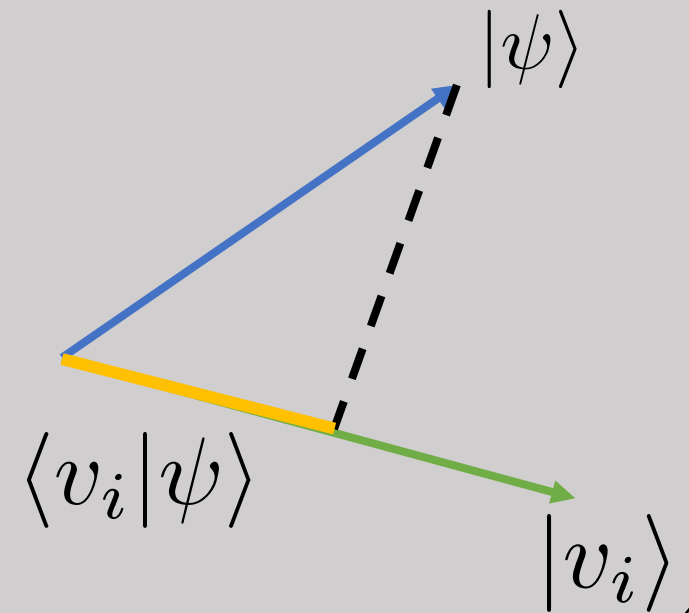
$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Measurement of orthonormal basis

Any orthonormal basis $\{|v_i\rangle\}$ that span \mathcal{H} has an associated measurement

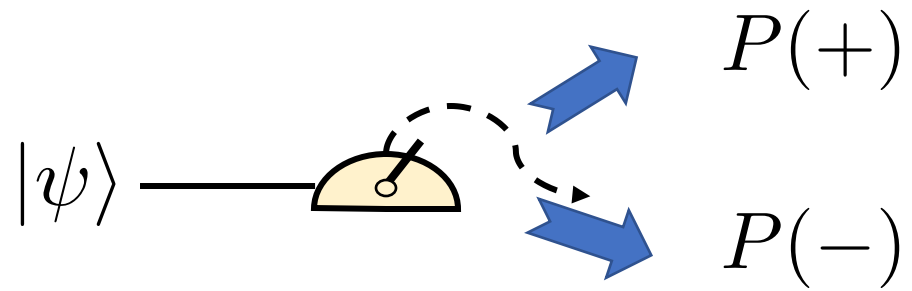
Probability of outcome i reads: $P(i) = |\langle v_i | \psi \rangle|^2$

The quantum state is updated to $|v_i\rangle$



Example: +/- basis

- $|\psi\rangle = \psi_0|0\rangle + \psi_1|1\rangle$



$$\mathcal{H}_{\mathcal{Q}} = \text{Span}\{|+\rangle, |-\rangle\}$$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

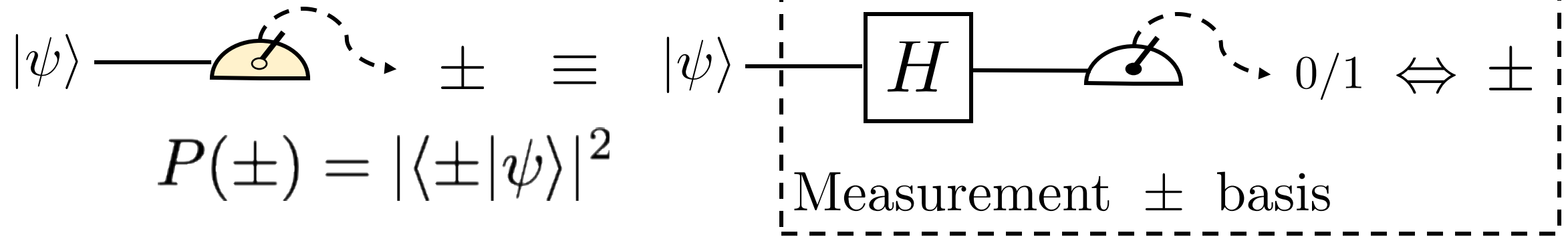
$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

- $P(+)=|\langle+|\psi\rangle|^2=|\frac{1}{\sqrt{2}}(\langle 1|+\langle 0|)(\psi_0|0\rangle+\psi_1|1\rangle)|^2=|\psi_0+\psi_1|^2/2$

- $P(-)=|\langle-|\psi\rangle|^2=|\psi_0-\psi_1|^2/2$

Arbitrary basis measurement

- Measurement basis $\{|\pm\rangle\}$:



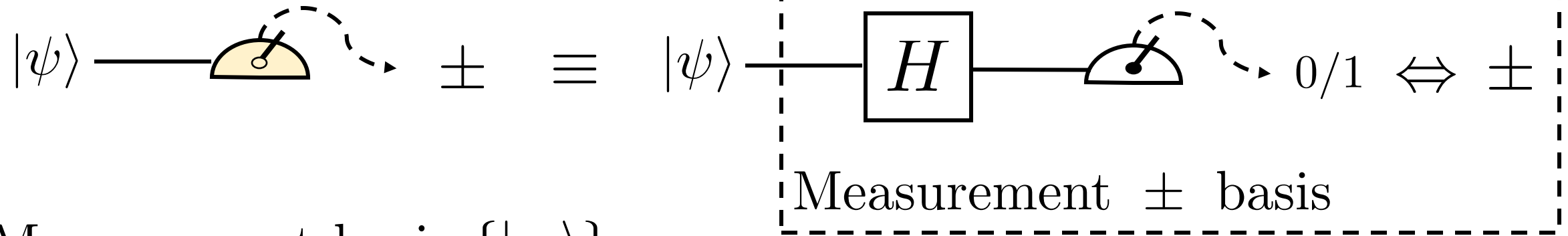
$$\begin{aligned} |+\rangle &= H|0\rangle \\ |-\rangle &= H|1\rangle \\ (A|\psi\rangle)^\dagger &= \langle\psi|A^\dagger \\ H^\dagger &= H \end{aligned}$$



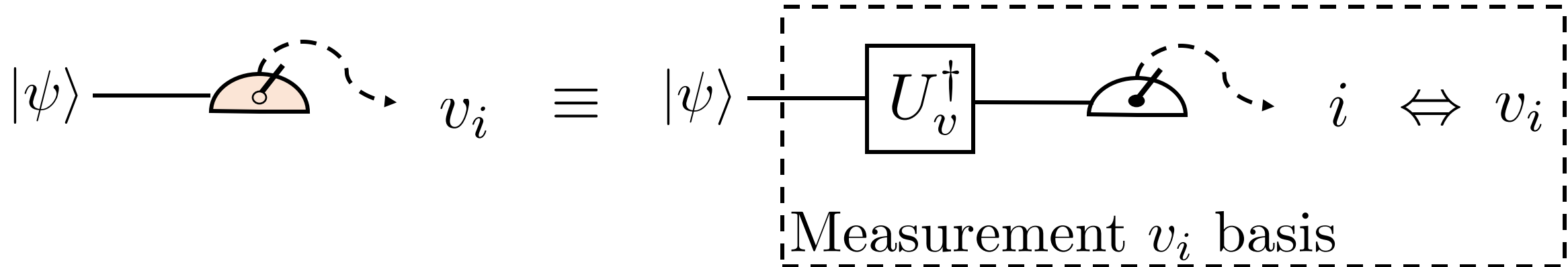
$$\begin{aligned} \langle + | \psi \rangle &= \langle 0 | H | \psi \rangle \\ \langle - | \psi \rangle &= \langle 1 | H | \psi \rangle \end{aligned}$$

Arbitrary basis measurement

- Measurement basis $\{|\pm\rangle\}$:



- Measurement basis $\{|v_i\rangle\}$:



$$\forall \text{ basis } \{|v_i\rangle\}, \exists U_v \text{ s.t. } |v_i\rangle = U_v |i\rangle \qquad \langle v_i | \psi \rangle = \langle i | U_v^\dagger | \psi \rangle$$

References

Reading references

1. NC 2.2.3 and 2.2.5

NC \equiv Michael Nielsen and Isaac Chuang, Quantum Computing and Quantum Information
Cambridge University Press (2010)

Lecture 10 we will revisit the measurement in terms of projectors

There is even a more general concept of measurement in N&C called POVM (2.2.6) that we will not cover in this course.

Even more general is the concept of [quantum instrument](#).

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Information-theoretic bounds on quantum advantage in machine learning

Hsin-Yuan Huang, Richard Kueng, John Preskill

Jan 08 2021 quant-ph cs.IT cs.LG math.IT arXiv:2101.02464v2

We study the performance of classical and quantum machine learning (ML) models in predicting outcomes of physical experiments. The experiments depend on an input parameter x and involve execution of a (possibly unknown) quantum process \mathcal{E} . Our figure of merit is the number of runs of \mathcal{E} required to achieve a desired prediction performance. We consider classical ML models that perform a measurement and record the classical outcome after each run of \mathcal{E} , and quantum ML models that can access \mathcal{E} coherently to acquire quantum data; the classical or quantum data is then used to predict outcomes of future experiments. We prove that for any input distribution $\mathcal{D}(x)$, a classical ML model can provide accurate predictions on average by accessing \mathcal{E} a number of times comparable to the optimal quantum ML model. In contrast, for achieving accurate prediction on all inputs, we prove that exponential quantum advantage is possible. For example, to predict expectations of all Pauli observables in an n -qubit system ρ , classical ML models require $2^{\Omega(n)}$ copies of ρ , but we present a quantum ML model using only $\mathcal{O}(n)$ copies. Our results clarify where quantum advantage is possible and highlight the potential for classical ML models to address challenging quantum problems in physics and chemistry.

Scited 156

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Fault-Tolerant Operation of a Quantum Error-Correction Code

Laird Egan, Dripto M. Debroy, Crystal Noel, Andrew Risinger, Daiwei Zhu, Debopriyo Biswas, Michael Newman, Muyuan Li, Kenneth R. Brown, Marko Cetina, Christopher Monroe

Sep 25 2020 quant-ph arXiv:2009.11482v2

Quantum error correction protects fragile quantum information by encoding it into a larger quantum system

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Blake Stacey 3 days ago

I'd had some notes kicking around a bit before someone mentioned that *Helgoland* had been translated into English and said some pertinent things. Maybe the book was a nudge for them, too.

Assessing Relational Quantum Mechanics

Wojciech Kryszak 7 days ago

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