Introduction to Quantum Programming and Semantics

Lecture 10: ZX Calculus

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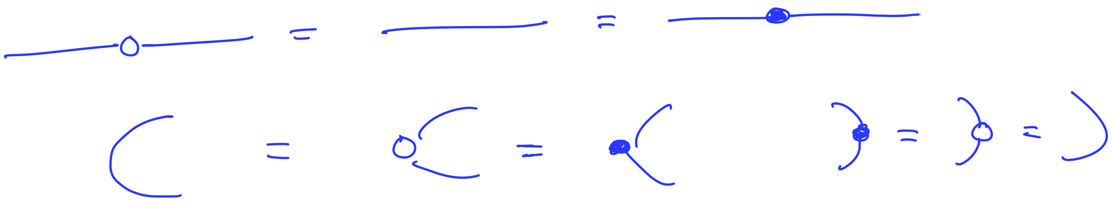
Overview

- ZX Calculus
- Soundness
- Completeness
- Quantum circuits
- Automation

ZX calculus

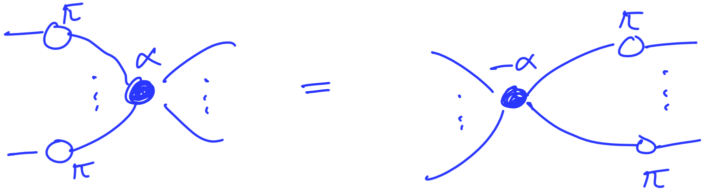
Rules

• wire rule:



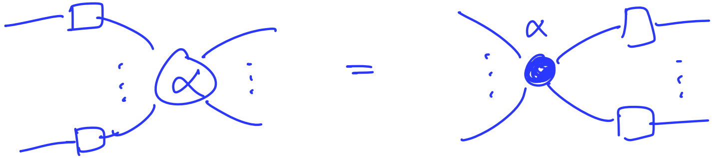
• spider fusion:

• π-rule:

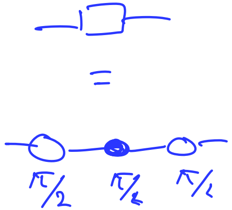


Rules

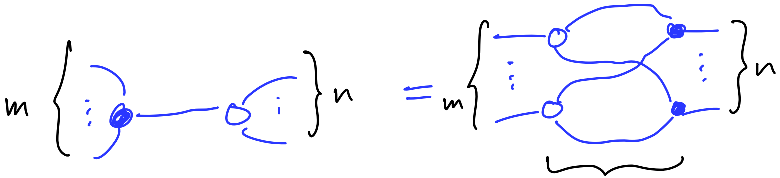
- colour change:



where

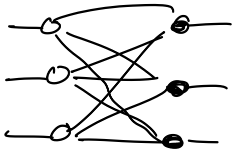


- strong complementarity

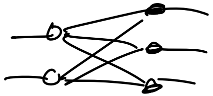


complete m,n-bipartite graph

e.g.



m = n = 3



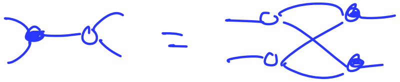
m = 2 n = 3

special cases: m = 0 ⟹

n = 0 ⟶

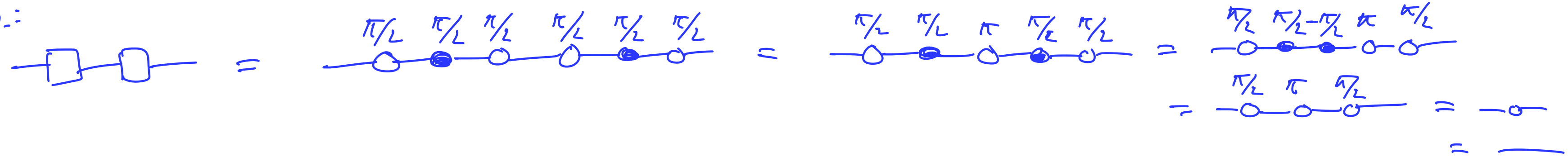
"copy rules"

m = n = 2 ⟶



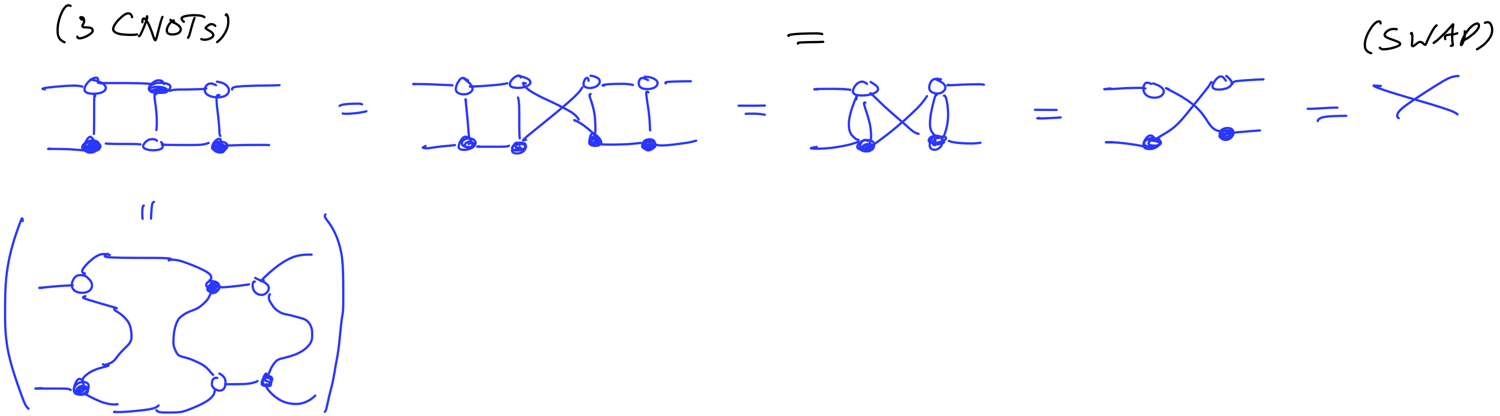
"bialgebra rule"

e.g.:



Lem:

e.g.:

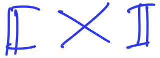


Soundness

Semantics

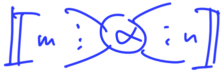
Soundness: if two ZX diagrams d, d' are equal then the matrices $[\![d]\!]$, $[\![d']\!]$ are equal

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$



$$= |\underbrace{0 \cdots 0}_{n}\rangle\langle\underbrace{0 \cdots 0}_{m}| + e^{i\alpha} |\underbrace{1 \cdots 1}_{n}\rangle\langle\underbrace{1 \cdots 1}_{m}| = \overbrace{\begin{pmatrix} 1 & & & \\ & 0 & & 0 \quad 0 \\ & & 0 \quad 0 & \\ 0 & 0 & & e^{i\alpha} \end{pmatrix}}^{2^m} \Bigg\} 2^n$$

$$= |+ \cdots +\rangle\langle + \cdots +| + e^{i\alpha} |- \cdots -\rangle\langle - \cdots -|$$

Completeness

Universality

thm: any n-qubit unitary can be constructed using only single qubit gates and CNOT gates

cor: any n-qubit unitary can be constructed as a ZX diagram

Approximate universality

<u>thm</u>: for any n qubit unitary U and $\varepsilon > 0$,

there exists a ZX diagram d s.t.:

- $\| U - d \| < \varepsilon$
- d only uses phases multiple of $\pi/8$

Completeness

thm: if d, d' are ZX diagrams, then:

matrices $[\![d]\!] = [\![d']\!]$ are equal

$\Longrightarrow$

$\exists$ legal rewrite from d to d'

needs: $\forall \alpha, \beta, \gamma \; \exists \alpha', \beta', \gamma'$:

↳ in terms of sin, cos of α, β, γ

Automation

Quantomatic, PyZX, QuiZX, ZX live

Summary:

- ZX calculus is sound and complete
- ZX calculus is universal
- $\pi/4$-ZX calculus is approximately universal
- ZX rewriting can be automated