Introduction to Quantum Programming and Semantics

Lecture 12: Measurement

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Overview

- Q#
- Measurement
- Mixed states
- Quantum channels
- Environment structures

Q#

Microsoft QDK

- Q# language

- Q# libraries with several standard operations and algorithms

- Dot.net integration with classical languages (Python, C#, F#, etc)

- Orchestration language to execute Q#:
    - Simulators
    - Resource estimators
    - Microsoft Azure quantum hardware

Q#

- Language itself is imperative …

- … but can be used in functional way through dot.net

- Not (necessarily) circuit description language:
  quantum instructions are dispatched in order
  and you can use measurement results in rest of program

  (cf measurement-based quantum computing, quantum error correction)

Measurement

Measurement

is only way to get info out of quantum system

for state $|\psi\rangle$, will tell you:
- the probability of outcome j
- the post-measurement state.

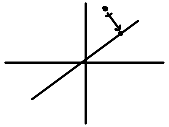
a <u>projection</u> is a linear map $\mathbb{C}^n \xrightarrow{P} \mathbb{C}^n$ s.t. $P = P^\dagger = P^2$

self-adjoint, idempotent

splits $\mathbb{C}^n$ into two pieces:
- $\mathrm{im}(P) = \{|\psi\rangle \mid P|\psi\rangle\}$
- $\mathrm{im}(P)^\perp = \{|\psi\rangle \mid P|\psi\rangle = 0\} = \mathrm{im}(1-P)$

$$\mathbb{C}^n = \mathrm{im}(P) \oplus \mathrm{im}(1-P)$$

e.g. $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$



$H \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} H$

Measurement

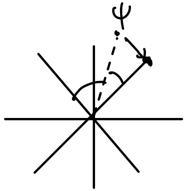
more generally, $P_1, \dots, P_k$, where $\sum_{j=1}^{k} P_j$ splits into k orthogonal pieces

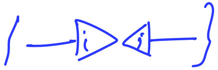
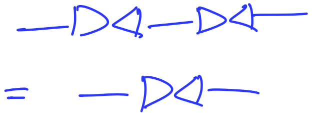
<u>von Neumann measurement</u> $\mathcal{M} = \{P_1, \dots, P_k\}$ s.t. $\sum_{j=1}^{k} P_j = 1$

probability of outcome j when measure state $|\psi\rangle$ with $\mathcal{M}$ is

$$\mathrm{Prob}(j \mid |\psi\rangle) = \langle \psi | P_j | \psi \rangle$$ — "Born rule"

note: if $|\psi\rangle = e^{i\theta} |\varphi\rangle$ then $\langle \psi | P_j | \psi \rangle = \langle e^{i\theta} \varphi | P_j | e^{i\theta} \varphi \rangle$



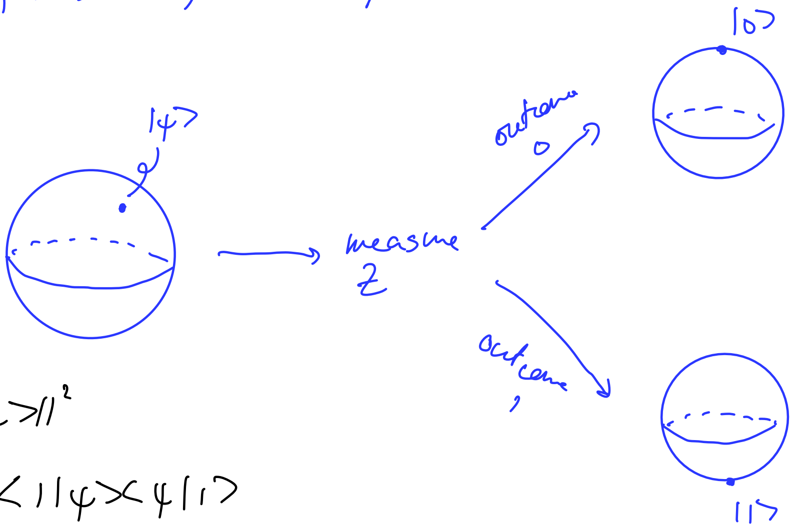
if  orthonormal basis, then  : 

and $\mathrm{Prob}(j \mid |\psi\rangle) = \langle \psi | P_j | \psi \rangle = \langle \psi | j \rangle \langle j | \psi \rangle = \| \langle j | \psi \rangle \|^2$

Measurement

after measurement: $|\psi\rangle \xrightarrow{\text{project}}$  $\xrightarrow{\text{normalise}} \frac{1}{\|P_j|\psi\rangle\|} \cdot$ 

e.g. $Z = \{ |0\rangle\langle 0| , |1\rangle\langle 1| \}$



with probability $\|\langle 0|\psi\rangle\|^2$

with probability $\|\langle 1|\psi\rangle\|^2$

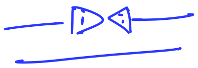
$\|\langle 0|\psi\rangle\|^2 + \|\langle 1|\psi\rangle\|^2$

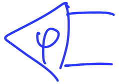
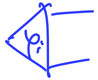
$= \langle 0|\psi\rangle\langle\psi|0\rangle + \langle 1|\psi\rangle\langle\psi|1\rangle$

$= \ldots$ write $\psi = \alpha|0\rangle + \beta|1\rangle \ \ldots \ = 1$

Mixed states

Mixed states

measure subsystem only: $M = \{$  $\}$

 $\overset{\text{outcome } i}{\rightsquigarrow}$ $\frac{1}{\| |\varphi_i\rangle \|}$  , where  $=$ 

if qubits not entangled, $\varphi = \psi_1 \otimes \psi_2$, then post-meas state $|i\rangle \otimes |\psi_2\rangle$

if qubits are entangled, post-meas $\sum_i |i\rangle \otimes |\varphi_i\rangle$

mixed state = convex combination of pure states

density matrix = matrix positive semidefinite, trace 1

$= \sum_{i=1}^{n} p_i |i\rangle\langle i|$ for some probabilities p_i, onb $|i\rangle$

Quantum channels

Completely positive maps

$$f: \mathbb{C}^{n^2} \longrightarrow \mathbb{C}^{m^2}$$

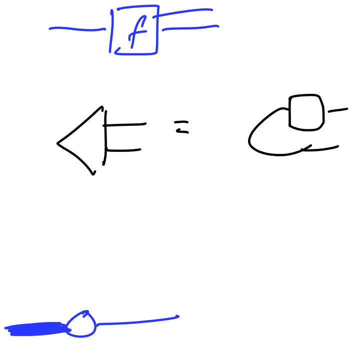
$$f(\rho) = \sum_i g_i^\dagger \rho g_i$$

$$C = \sum_i |ii\rangle$$

Environment structures

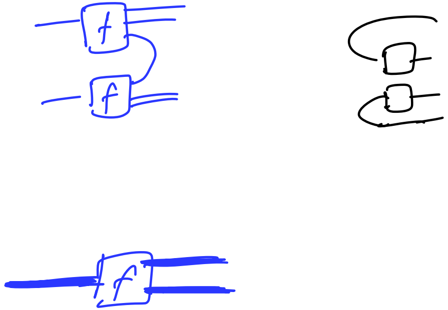
Thick vs thin wires

pure state picture



$\rightsquigarrow$

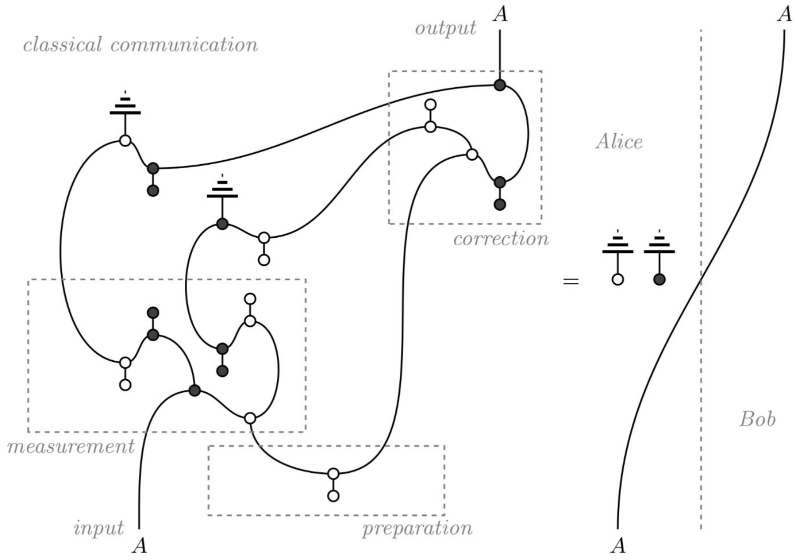
mixed state picture

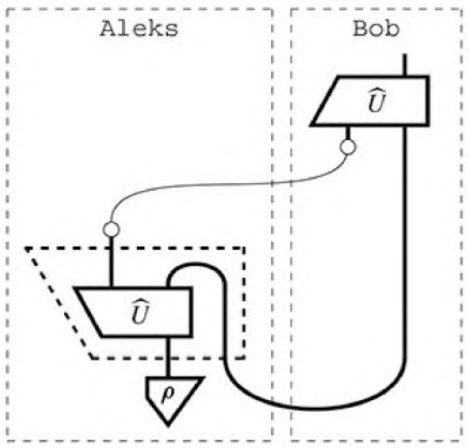


thick wire carries qubit

thin wire carries bits

Quantum teleportation





Summary:

- Q#: not necessarily circuit description, mid-circuit measurement
- Mixed states: partial knowledge
- Measurement: postselection, sums, graphically