Introduction to Quantum Programming and Semantics

Lecture 15: Quantum simulation

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Overview

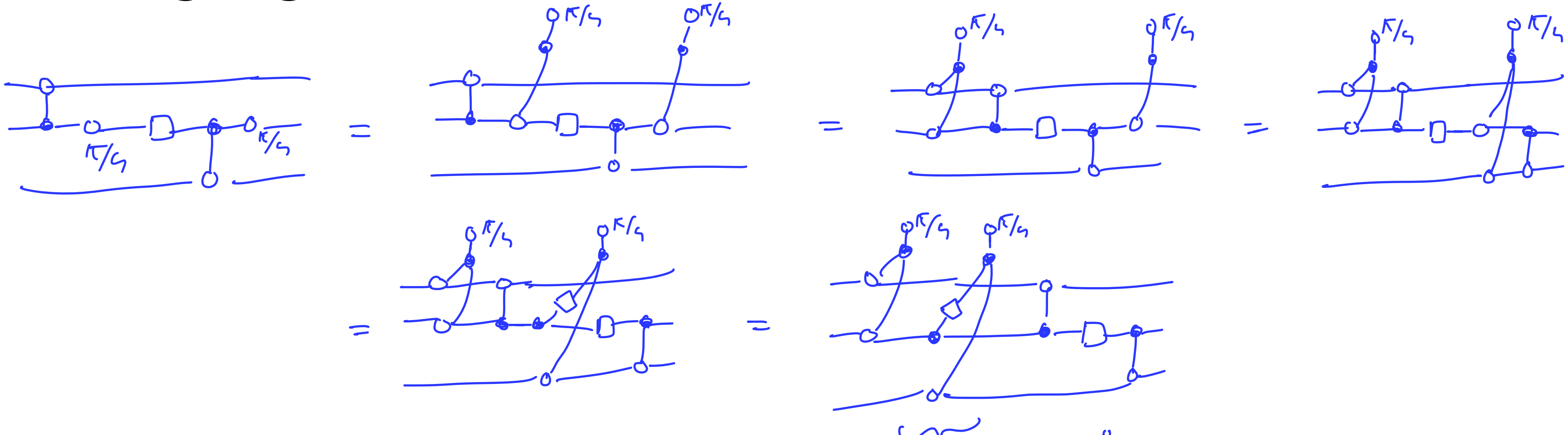
- Pauli gadgets
- Hamiltonian simulation

Pauli gadgets

Pauli gadgets

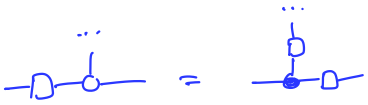
Clifford + phase is universal gate set

Q: can move all non-Clifford phases out?

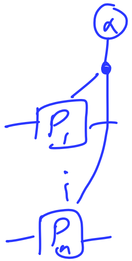


"Pauli gadget"

e.g. H gates :

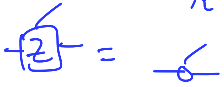


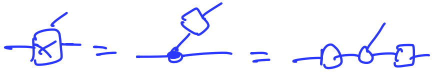
Pauli gadgets

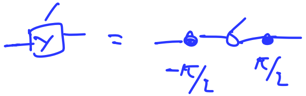
Prop: If $P_i \in \{I, X, Y, Z\}$ and $\vec{P} = P_1 \otimes \cdots \otimes P_n$, then the map 

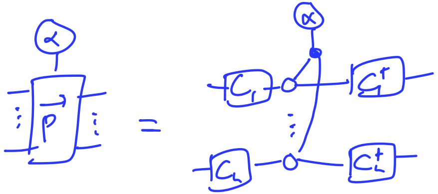
is unitary:

where



Pf: Note 

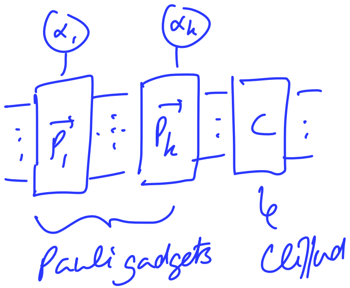
and 

So 

for Cliffords C_i. $\boxtimes$

Pauli gadgets

Thm any Clifford + phase circuit can be written as:

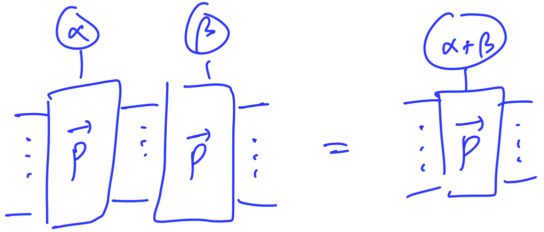


Pf idea: - show Clifford gates commute past Pauli gadgets

- move phases out of C.

Pauli gadgets

can fuse:



and if $\vec{P}\vec{Q} = \vec{Q}\vec{P}$ then $\vec{P}(\alpha)\vec{Q}(\beta) = \vec{Q}(\beta)\vec{P}(\alpha)$

algorithm to extract circuits:

1. compute Pauli gadget form of circuit
2. commute Pauli gadgets and combine phase where possible
3. merge Pauli gadgets with Clifford phases into Clifford part.
4. repeat.
5. extract circuit (many options)

Hamiltonian simulation

self-adjoint matrix

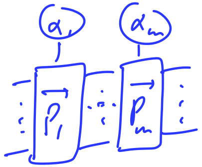
Hamiltonian simulation

if $|\psi_t\rangle = e^{-itH}|\psi_0\rangle$

Q: design circuit that implements e^{-itH}

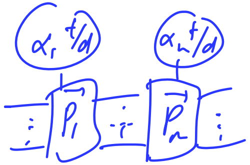
observation: Paulis span space of self-adjoint matrices: $H = \frac{1}{2}\sum_{j=1}^{m} \alpha \vec{P}_j$

if all $\vec{P}_j$ commute, then easy: $U = e^{-itH} = e^{-it\alpha_1\vec{P}_1} \cdots e^{-it\alpha_m\vec{P}_m}$

$U(t) =$ 

if don't commute, not true, but can still approximate

trick: make t very small $\qquad U = e^{-itH} = \left(e^{-it/d \cdot H}\right)^d \qquad$ "Trotterisation"

$U(t/d)^d \underset{d\cdot\left(\frac{t}{d}\right)^2}{\approx}$ 

error $d\left(t/d\right)^2 = t^2/d \longrightarrow 0$ as $d \to \infty$

Summary:

- ZX diagrams with phases pi/4 are fully universal but difficult
- Pauli gadgets can graphically handle exponentials of Pauli matrices
- Can simulate Hamiltonians by chopping into small time steps