Introduction to Quantum Programming and Semantics

Lecture 4: Graphical calculus

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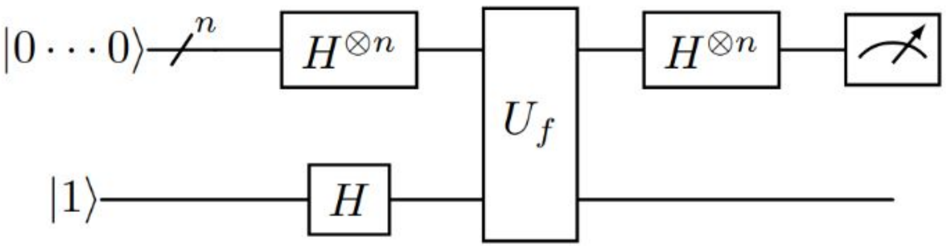
Overview

- String diagrams
- States, effects, scalars
- Entanglement

Deutsch-Jozsa



$$|01\rangle \xmapsto{H\otimes H} \frac{1}{2}\begin{pmatrix}1&1\\1&-1\end{pmatrix}\otimes\begin{pmatrix}1&1\\1&-1\end{pmatrix} \;|01\rangle$$

$$= \frac{1}{2}\begin{pmatrix}1&1&1&1\\1&-1&1&-1\\1&1&-1&-1\\1&-1&-1&1\end{pmatrix} |01\rangle \;=\; \frac{1}{2}\left(|00\rangle-|01\rangle+|10\rangle-|11\rangle\right)$$

$$= \frac{1}{2}\left(|0\rangle(|0\rangle-|1\rangle)+|1\rangle(|0\rangle-|1\rangle)\right)$$

$$= \frac{1}{\sqrt{2}^{\,n+1}}\sum_{x=0}^{2^n-1} x\left(|0\rangle-|1\rangle\right)$$

$$\xmapsto{U_f} \frac{1}{2}\left(|0\rangle(|0\oplus f_0\rangle-|1\oplus f_0\rangle)+|1\rangle(|0\oplus f_1\rangle-|1\oplus f_1\rangle)\right)$$

$$= \frac{1}{\sqrt{2}^{\,n+1}}\sum_{x} x\left(|0\oplus f(x)\rangle-|1\oplus f(x)\rangle\right)$$

$$= \frac{1}{2}\left(|0\rangle(-1)^{f_0}(|0\rangle-|1\rangle)+|1\rangle(-1)^{f_1}(|0\rangle-|1\rangle)\right)$$

$$\xmapsto{\text{ignore last}} \frac{1}{\sqrt{2}}\left(|0\rangle\cdot(-1)^{f_0}+|1\rangle(-1)^{f_1}\right)$$

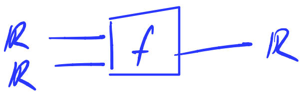
$$= \sum_{y=0}^{2^n-1}\left(\frac{1}{2^n}\sum_{x=0}^{2^n-1}(-1)^{fx}(-1)^{xy}\right)|y\rangle$$

$$\xmapsto{H} \frac{1}{2}$$

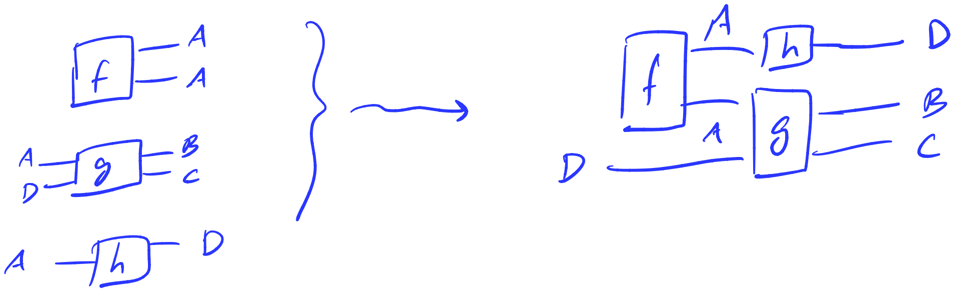
$$\xmapsto{|00\rangle} \left|\frac{1}{2}\left((-1)^{f_0}+(-1)^{f_1}\right)\right|$$

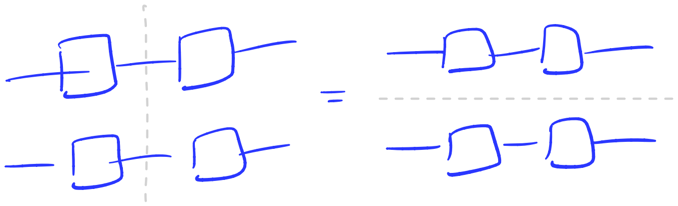
String diagrams

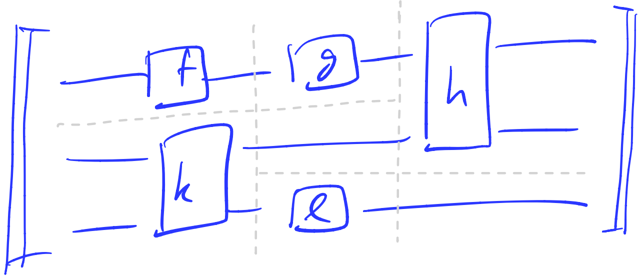
String diagrams



$f(x,y) = x^2 + y$

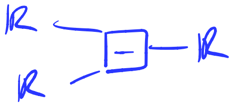


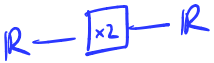


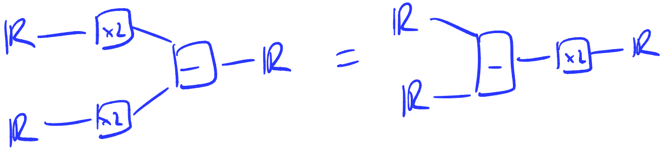


$$= (⟦h⟧ \otimes id) \circ (⟦g⟧ \otimes id \otimes ⟦\ell⟧) \circ (⟦f⟧ \otimes ⟦k⟧)$$

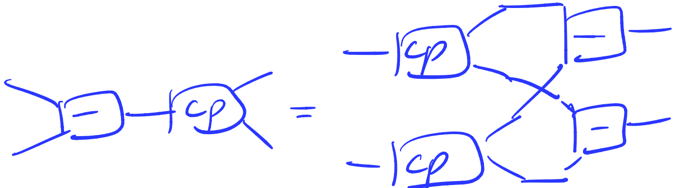
Diagrammatic reasoning





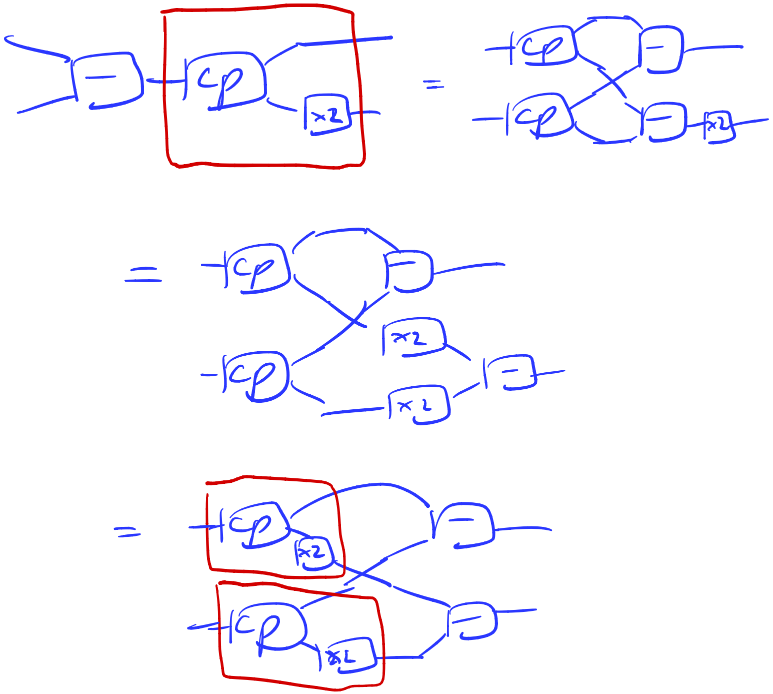


$x \longmapsto (x,x)$

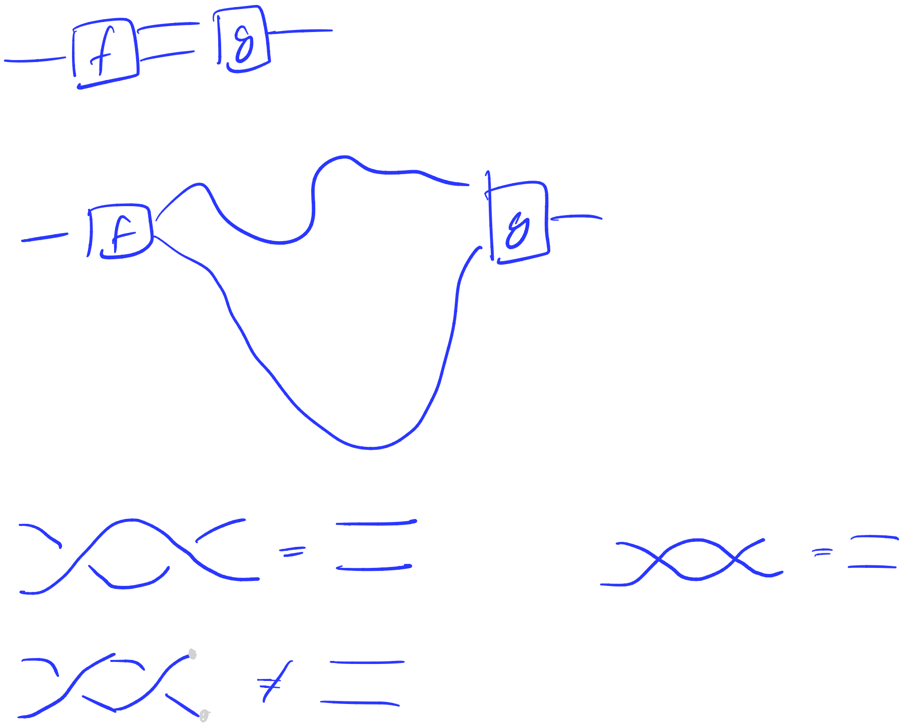


$(x,y) \longmapsto (x,x,y,y)$

$\longmapsto (x,y,x,y) \longmapsto (x-y,x-y)$



Isotopy

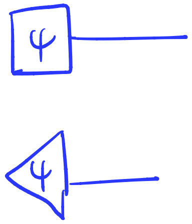


Soundness and completeness

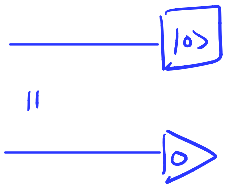
algebra	pictures
$\otimes$, $\circ$	2d, no X
$\otimes$, $\circ$, braided (topological q. comp.)	3d, 
$\otimes$, $\circ$, symmetry	4d, X

States, effects, scalars

States



Effects



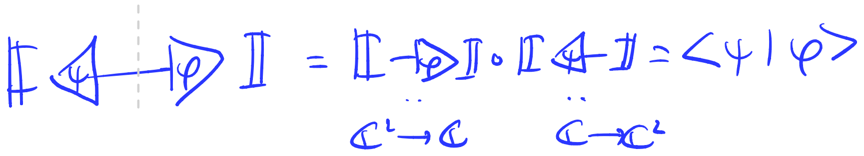
postselection

(Sila

Scalars

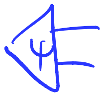
$$\left[\!\left[\boxed{\alpha} \right]\!\right] : \mathbb{C}^{(2^0)} = \mathbb{C} \longrightarrow \mathbb{C}^{(2^0)} = \mathbb{C}$$

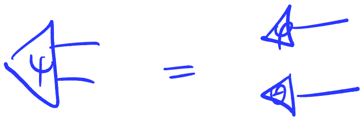
$$\boxed{\alpha}\ \boxed{\beta} = \begin{array}{c}\boxed{\alpha}\\ \boxed{\beta}\end{array} = \boxed{\beta}\ \boxed{\alpha}$$



Entanglement

Entanglement

bipartite state is of form 

it is separable if  for some states φ, θ

it is entangled otherwise

Summary:

- String diagrams have (4d) graphical calculus
- Sound and complete for sequential and parallel composition
- States, effect, and scalars are derived concepts
- Entanglement makes sense at graphical level

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