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Introduction to Quantum Programming and Semantics

Lecture 5: Bending space and time

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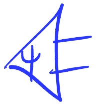
Overview

- Cups and caps
- Quantum teleportation
- Map-state duality
- Graphical symmetries

Cups and caps

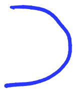
Bell states

$$\begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix} =$$  $\neq$  $$= \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \in \mathbb{C}^4 = \mathbb{C}^2 \otimes \mathbb{C}^2$$ is called Bell state

is maximally entangled

special notation  also called "cup"

transpose $\begin{pmatrix} 1 & 0 & 0 & 1 \end{pmatrix} : \mathbb{C}^4 \longrightarrow \mathbb{C}^1$

special notation  also called "cap"

Snake equation

syntax

semantics

$$\|$$

$$(\mathrm{Bell}^T \otimes id) \circ (id \otimes \mathrm{Bell})$$

$$\left.\begin{array}{l} \mathbb{C}^2 \xrightarrow{id_{\mathbb{C}^2}} \mathbb{C}^2 \\ \mathbb{C}^1 \xrightarrow{\mathrm{Bell}} \mathbb{C}^4 \end{array}\right\} \Longrightarrow \mathbb{C}^2 \otimes \mathbb{C}^1 \xrightarrow{id \otimes \mathrm{Bell}} \mathbb{C}^2 \otimes \mathbb{C}^4$$

$$\mathbb{C}^2 \simeq \mathbb{C}^2 \otimes \mathbb{C}^1 \xrightarrow{id \otimes \mathrm{Bell}} \mathbb{C}^2 \otimes \mathbb{C}^4 = \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2 \simeq \mathbb{C}^4 \otimes \mathbb{C}^2 \xrightarrow{\mathrm{Bell}^T \otimes id} \mathbb{C}^1 \otimes \mathbb{C}^2 \simeq \mathbb{C}^2$$

Graphical calculus of string diagrams including cups and caps

is still sound and complete

with nondirected isotopy:

graphs G, H embedded in $\mathbb{R}^4$ isotopic

$\Longleftrightarrow$

$[0,1] \times \mathbb{R}^4 \xrightarrow{\varphi} \mathbb{R}^4$ s.t.:

$\varphi(0) = G$

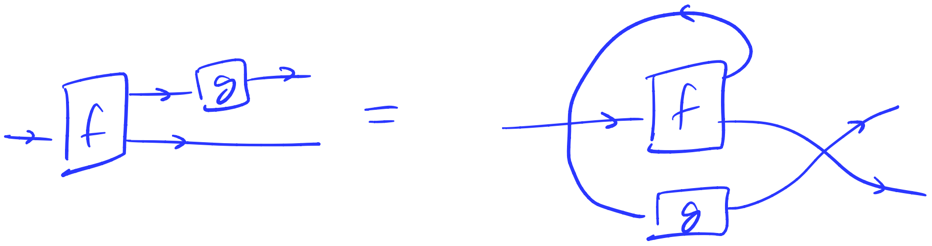
$\varphi(1) = H$

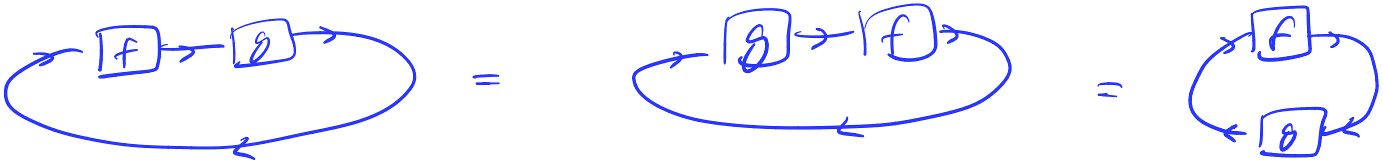
φ smooth

$\forall t \in [0,1]: \varphi(t)(\overset{\text{input/output}}{\text{vertex } v}) = v$

$\cancel{\forall t \in [0,1]: s \leq s' \Rightarrow \varphi(t)(s, x, y, z)_1 \leq \varphi(t)(s', x, y, z)_1}$

still: only connectivity matters





Teleportation

Quantum teleportation

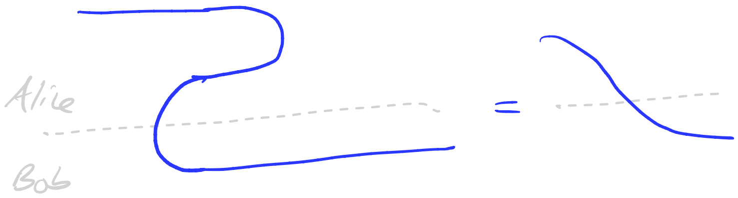
Alice and Bob share Bell state

Alice gets unknown input qubit $|\psi\rangle$

goal: Bob has $|\psi\rangle$

method:

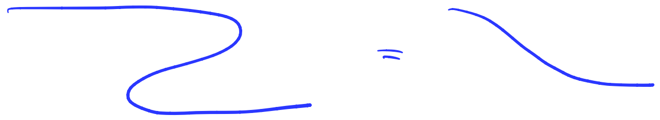
1. Alice measures her half and $|\psi\rangle$
2. A sends outcome to B; 2 bits
3. B applies unitary to his half



Teleportation in OpenQASM

```
gate H a { U ( pi /2 , 0 , pi ) a ; }
gate X a { U ( pi , 0 , pi ) a ; }
gate Z a { U (0 , 0 , pi ) a ; }
gate CX a , b { ctrl @ U ( pi , 0 , pi ) a , b ; }

qubit input_state ;
reset input_state ;

// Create a Bell state
qubit [2] bell_state ;
reset bell_state ;
H bell_state [0];
CX bell_state [0] , bell_state [1];

// Entangle the input state with Bell state
CX input_state , bell_state [0];

// Measure and correct
bit m1 = measure input_state ;
bit m2 = measure bell_state [0];
if ( m2 == true ) { X bell_state [1]; }
if ( m1 == true ) { Z bell_state [1]; }
```

"Classical teleportation"



A has a bit she wants to send to B

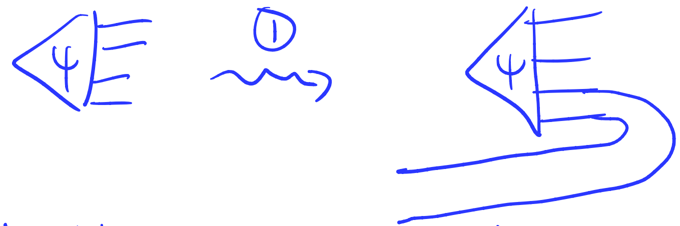
$$C = \begin{pmatrix} 1/2 \\ 0 \\ 0 \\ 1/2 \end{pmatrix}$$ share a random bit

or one-time pad

protocol now means one-time pad encryption

Taking names

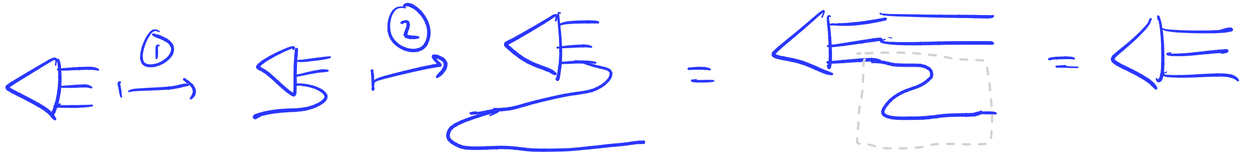
Names



'state' 'gate'

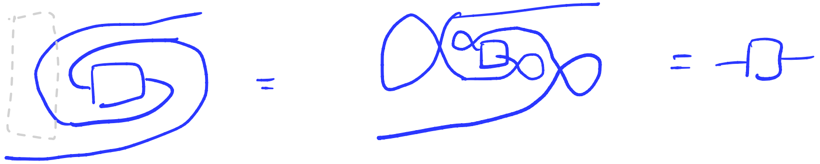
function "name"

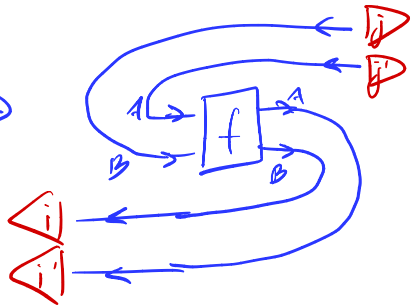
1-to-1 correspondence



Transpose and adjoint

Transpose

$f^{TT} =$  $= f$

$f : A \otimes B \longrightarrow A \otimes B \quad \Longrightarrow$ 

$: \; B^* \otimes A^* \longrightarrow B^* \otimes A^*$

"dual vector space"

$A = B = \mathbb{C}^2$

$f : \mathbb{C}^4 \longrightarrow \mathbb{C}^4$

$\parallel$

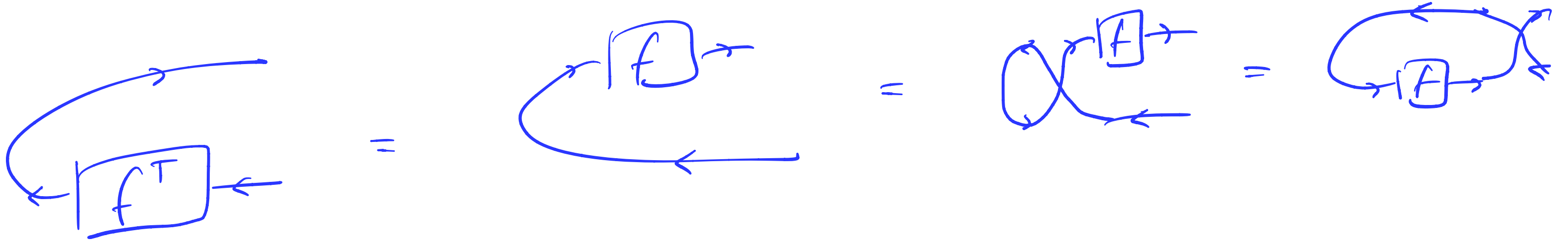
$f^T \leftarrow$ "transpose" $: \; \mathbb{C}^4 \longrightarrow \mathbb{C}^4$

$\langle j j' | f | i i' \rangle$

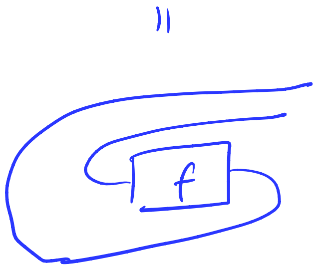
$\langle j j' | f^T | i i' \rangle = \langle i i' | f | j j' \rangle$

so f^T is transpose matrix of f

Claim:

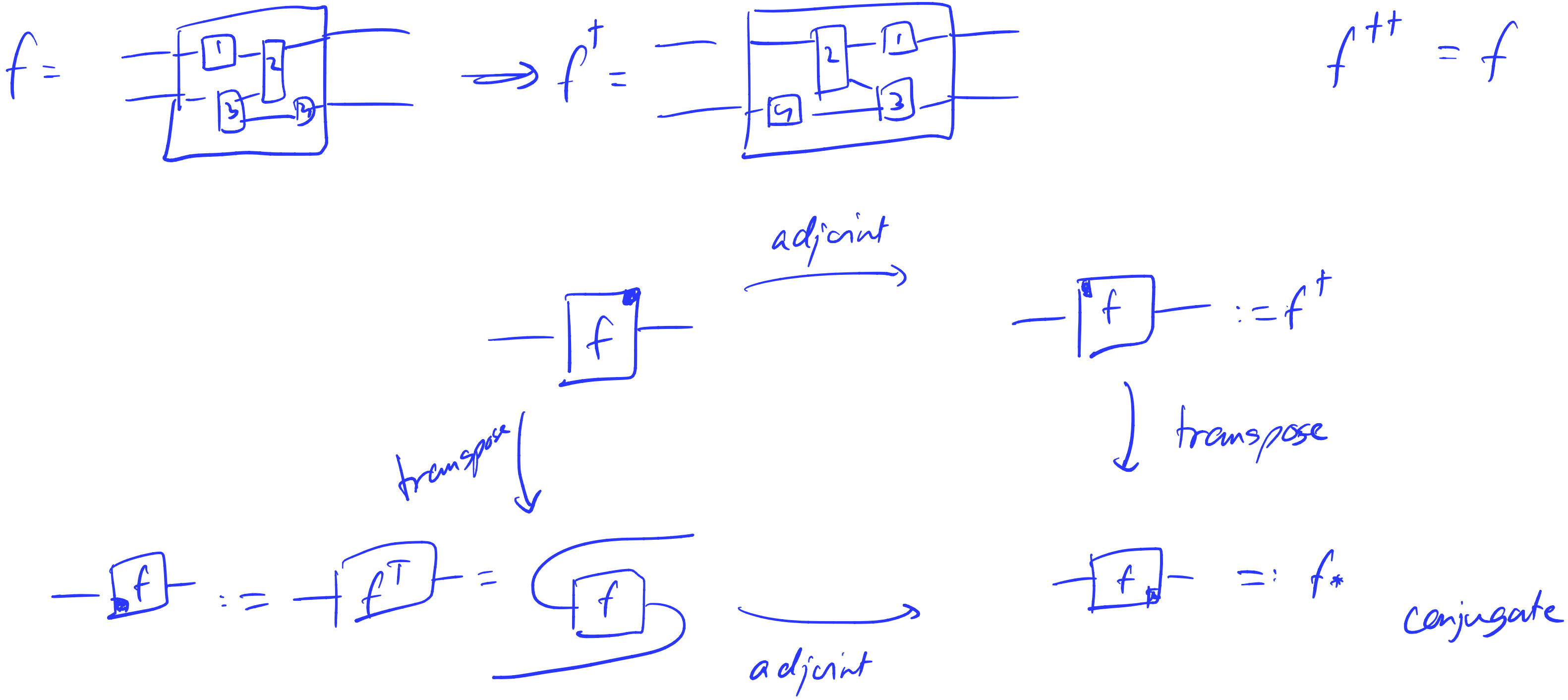


Proof:



Conclusion: can slide boxes along wires

Adjoint



Summary:

- Can trade time for space in string diagrams
- Models (postselected) quantum teleportation
- Can trade maps for states in string diagrams: gate injection
- Graphical calculus has meaningful symmetries