

Introduction to Quantum Programming and Semantics

Lecture 7: Copying and deleting

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Overview

- Copying and deleting
- No-cloning and no-deleting
- Products
- Categories

Copying and deleting

Copying machines

$$A \xrightarrow{d} A \otimes A$$

$$\boxed{d} = \text{circuit diagram with two outputs}$$

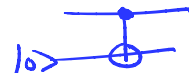
$$\text{circuit diagram with one output} = \text{circuit diagram with one output}$$

• commutative: $\boxed{d} \text{ with crossing wires} \stackrel{(1)}{=} \boxed{d}$

• associative: $\boxed{d} \text{ with } \boxed{d} \text{ on top} \stackrel{(2)}{=} \boxed{d} \text{ with } \boxed{d} \text{ on right}$

• unital: $\boxed{d} \text{ with } \text{triangle} \stackrel{(3)}{=} \text{line} (= \boxed{d} \text{ with } \text{triangle})$

Such a process d will a "copying machine" (for things of type A) or "comonoid"



e.g.: $d = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $|0\rangle|1\rangle$

$$(3) \Leftrightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = (e \otimes \text{id}) \circ d$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

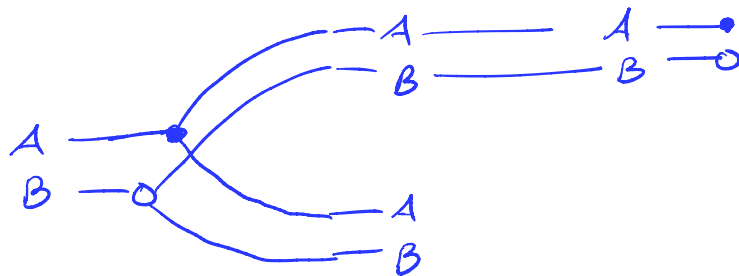
$$\mathbb{C} \xleftarrow{e} \mathbb{C}^2 \xrightarrow{d} \mathbb{C}^2 \otimes \mathbb{C}^2$$

$$1 \leftarrow |0\rangle \mapsto |00\rangle$$

$$1 \leftarrow |1\rangle \mapsto |11\rangle$$

$$e = \begin{pmatrix} 1 & 1 \end{pmatrix}$$

Combining comonoids



Monoids

$\begin{array}{c} A \\ A \end{array} \begin{array}{c} \diagup \\ \diagdown \end{array} \circ \begin{array}{c} \diagdown \\ \diagup \end{array} \begin{array}{c} A \\ A \end{array}$ s.t. associative

$\begin{array}{c} \diagup \\ \diagdown \end{array} \begin{array}{c} \diagup \\ \diagdown \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} = \begin{array}{c} \diagup \\ \diagdown \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} \begin{array}{c} \diagup \\ \diagdown \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array}$

$\circ \begin{array}{c} A \\ A \end{array}$ (commutative $\begin{array}{c} \diagup \\ \diagdown \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} = \begin{array}{c} \diagdown \\ \diagup \end{array} \begin{array}{c} \diagup \\ \diagdown \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array})$

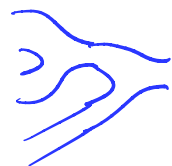
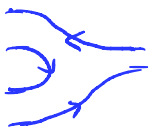
unital $\begin{array}{c} \circ \\ \diagdown \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} \begin{array}{c} A \\ A \end{array} = \text{---} = \begin{array}{c} \diagdown \\ \diagup \end{array} \begin{array}{c} \circ \\ \diagup \end{array} \begin{array}{c} A \\ A \end{array}$

Pairs of pants

e.g. $A = n\text{-by-}n \text{ matrices}$
 $= \{f: \mathbb{C}^n \rightarrow \mathbb{C}^n \mid \text{linear}\}$
 $\simeq (\mathbb{C}^n)^* \otimes \mathbb{C}^n$

$$f, g: \mathbb{C}^n \rightarrow \mathbb{C}^n$$

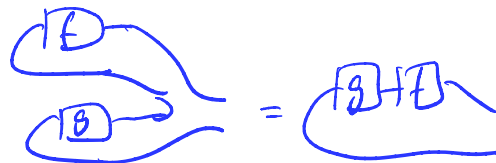
$$[f], [g]: \mathbb{C} \rightarrow \mathbb{C}^n \otimes \mathbb{C}^n$$



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No-cloning and no-deleting

Uniform deleting

Systematic deleting: for each type A , have $A \rightarrow 0$
s.t. for any f : $A \rightarrow 0 = A \rightarrow \boxed{f} \xrightarrow{B} 0$

$$\text{and: } A \otimes B \rightarrow 0 = \frac{A \rightarrow 0}{B \rightarrow 0}$$

Consequence: unique map $A \rightarrow I$

$$A \rightarrow \boxed{e}$$

$$A \rightarrow \boxed{e'}$$

Uniform copying

Copying systematically means:

- for each type A , choose $A \multimap A$ s.t.:

- for each map $f: A \rightarrow B$, $A \multimap [f]_B = [f]_B \multimap B$

- $A \otimes B \multimap C = A \multimap B \multimap C$

No-cloning theorem

Lem: $\frac{C^A}{C} = \alpha$

Pf: $\frac{C}{C} = \boxed{d_{\perp}} \frac{C}{C} = C \boxed{d_{\text{max}}} = \text{diagram} \quad (*)$

$\frac{C}{C} \stackrel{(*)}{=} \text{diagram} = \text{diagram} \stackrel{(*)}{=} \sum = \alpha$

Lem: $X = \equiv$

Pf: $X = \text{diagram} = \text{diagram} = \equiv$

Thm: If uniform copying,

then $\boxed{f} = \overline{\text{diagram}}$

Pf: $\boxed{f} = \text{diagram} = \overline{\text{diagram}} = \text{diagram}$
 $= \overline{\text{diagram}} = \text{diagram}$

Products

Universal property

Characterising products

Categories

Composition

Summary:

- Comonoids are copying machines
- No-cloning theorem holds syntactically
- Can recognise when semantics is classical via products
- Can interpret string diagrams generally in monoidal categories