Introduction to Quantum Programming and Semantics

Lecture 7: Copying and deleting

Chris Heunen

University of Edinburgh

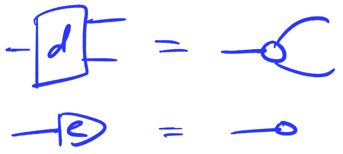
Overview

- Copying and deleting
- No-cloning and no-deleting
- Products
- Categories

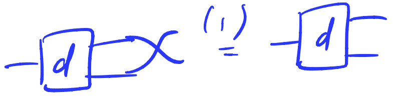
Copying and deleting

Copying machines

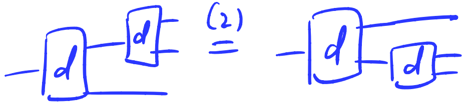
$$A \xrightarrow{d} A \otimes A$$



- commutative :

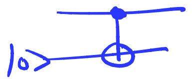


- associative :



- unital

Such a process d we'll a "copying machine" (for things of type A) or "comonoid"



e.g.:

$$d = \begin{array}{c} \begin{array}{cc} |0\rangle & |1\rangle \end{array} \\ \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \end{array} \; |00\rangle$$

$$e = \begin{pmatrix} 1 & 1 \end{pmatrix}$$

$$(3) \Longleftrightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = (e \otimes id) \circ d$$

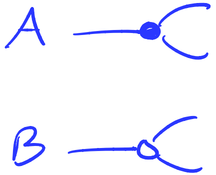
$$= \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$$

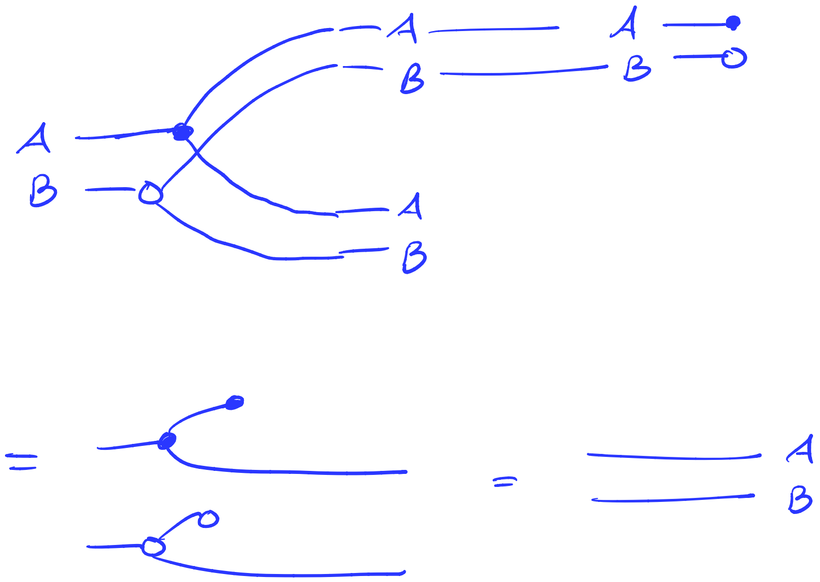
$$\mathbb{C} \xleftarrow{e} \mathbb{C}^2 \xrightarrow{d} \mathbb{C}^2 \otimes \mathbb{C}^2$$

$$1 \longleftarrow\!\shortmid\; |0\rangle \longmapsto |00\rangle$$

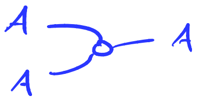
$$1 \longleftarrow\!\shortmid\; |1\rangle \longmapsto |11\rangle$$

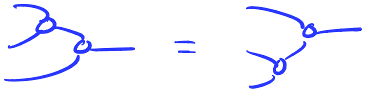
Combining comonoids





Monoids



s.t. associative 

(commutative )

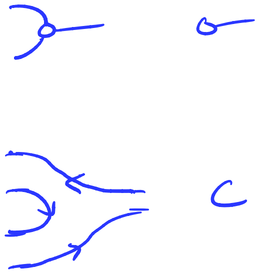
unital 

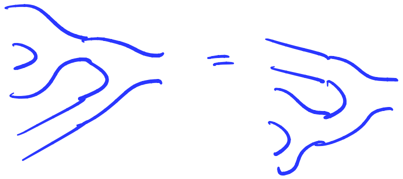
Pairs of pants

e.g. $A =$ n-by-n matrices

$= \{f : \mathbb{C}^n \to \mathbb{C}^n \text{ linear}\}$

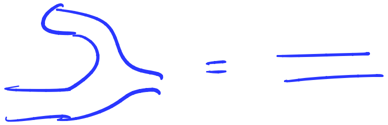
$\simeq (\mathbb{C}^n)^* \otimes \mathbb{C}^n$

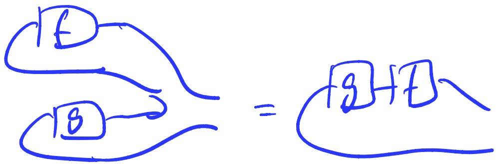




$f, g : \mathbb{C}^n \to \mathbb{C}^n$

$\ulcorner f \urcorner, \ulcorner g \urcorner : \mathbb{C} \to \mathbb{C}^{n*} \otimes \mathbb{C}^n$





No-cloning and no-deleting

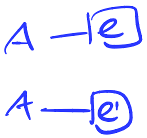
Uniform deleting

Systematic deleting: for each type A, have 

s.t. for any f: 

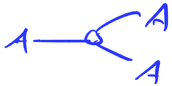
and: 

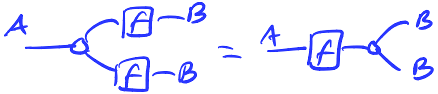
Consequence: unique map $A \longrightarrow I$

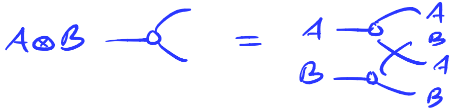


Uniform copying

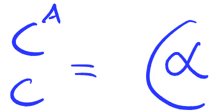
copying systematically means:

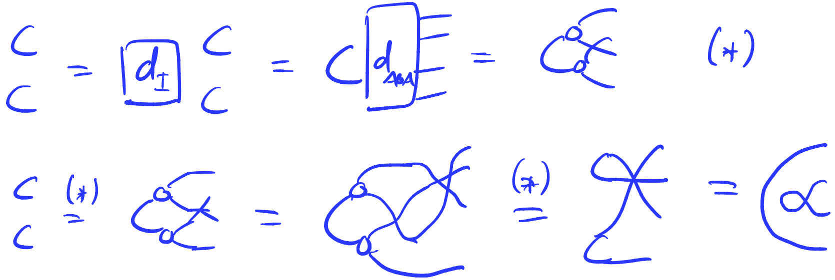
- for each type A, choose  s.t.:

- for each map f: 

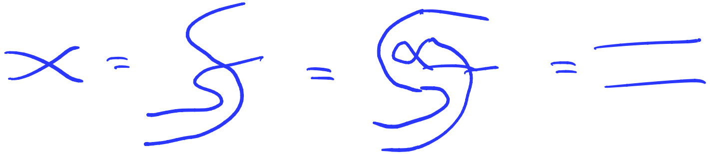
- 

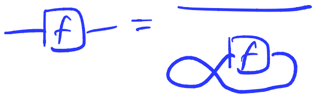
No-cloning theorem

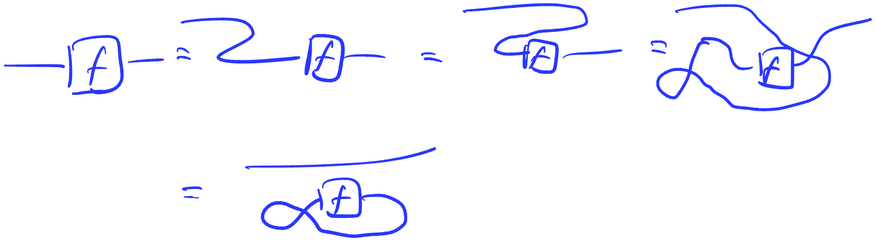
Lem: 

Pf: 

Lem: 

Pf: 

Thm: If uniform copying,
then 

Pf: 

Products

Universal property

Characterising products

Categories

Composition

Summary:

- Comonoids are copying machines
- No-cloning theorem holds syntactically
- Can recognise when semantics is classical via products
- Can interpret string diagrams generally in monoidal categories