# Introduction to Quantum Programming and Semantics

## Lecture 8: Classical data

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# Overview

- Qiskit
- Frobenius algebras
- Spiders
- Phases

# Frobenius algebras

# Copying orthonormal bases

$\{|0\rangle, |1\rangle\}$ basis for $\mathbb{C}^2$ $\rightsquigarrow$ copying machine $\quad -\!\!< \; : \mathbb{C}^2 \to \mathbb{C}^2 \otimes \mathbb{C}^2$

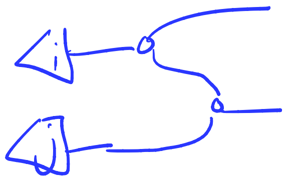
$$|0\rangle \mapsto |00\rangle$$
$$|1\rangle \mapsto |11\rangle$$

what is adjoint? $\quad >\!\!- \; : \mathbb{C}^2 \otimes \mathbb{C}^2 \to \mathbb{C}^2$ $\quad$ comparison

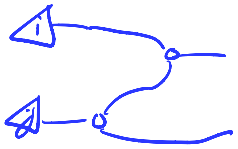
$$|ij\rangle \mapsto \begin{cases} |i\rangle & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

$$\langle -\!\!<(|i\rangle) \mid jk\rangle = \langle i \mid >\!\!-(|jk\rangle)\rangle$$
$$\|$$
$$\langle ii \mid jk\rangle$$
$$\|$$
$$\begin{cases} 1 & \text{if } i = j = k \\ 0 & \end{cases}$$

Idea:



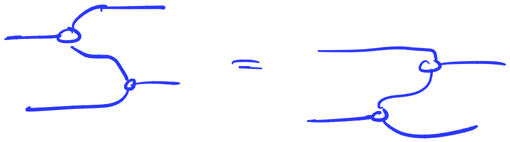
$$= \begin{bmatrix} |ii\rangle & \text{if } i=j \\ 0 & \text{if } i\neq j \end{bmatrix} =$$



# Interacting monoid and comonoid

in the example of copying a basis $|i\rangle \mapsto |ii\rangle$

Frobenius law:



basis is orthogonal

$\langle i|j\rangle = 0$ if $i \neq j$

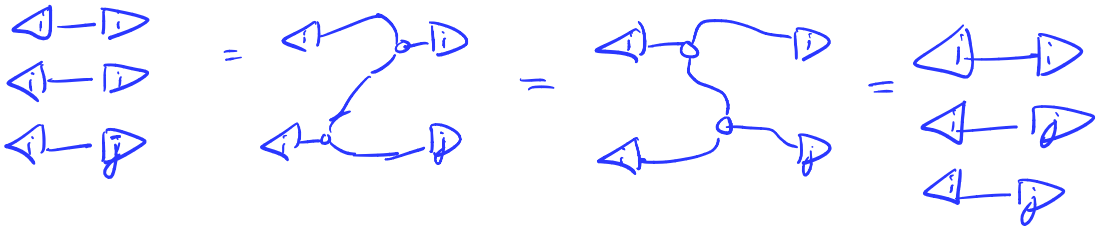
speciality:

basis vectors are normal

$\langle i|i\rangle = 1$

**Claim:** if copies basis, and satisfies Frobenius, then basis vectors are orthogonal.

**Pf:**



if $\langle i|j\rangle = 0$

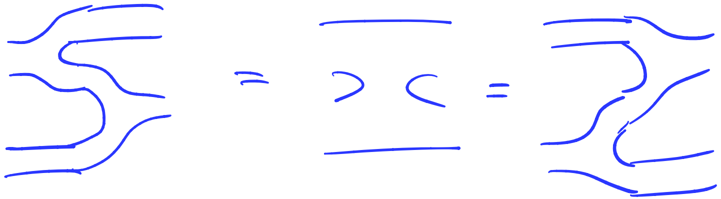
if $\langle i|j\rangle \neq 0$, then $\langle i|i\rangle = \langle i|j\rangle$, similarly $\langle j|i\rangle = \langle j|j\rangle$

so $\langle i-j|i-j\rangle = \langle i|i\rangle - \langle i|j\rangle - \langle j|i\rangle + \langle j|j\rangle = 0$

so $i-j=0$ so $i=j$.

Examples : • $|i\rangle \mapsto |ii\rangle$ for orthonormal basis $\{|i\rangle\}$

• pair of pants:



i.e. n-by-n matrices.

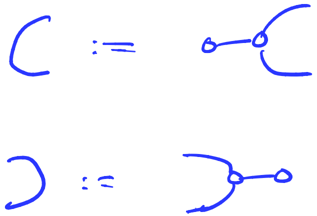
# Classical structure is:

a map  : $A \longrightarrow A \otimes A$ that:

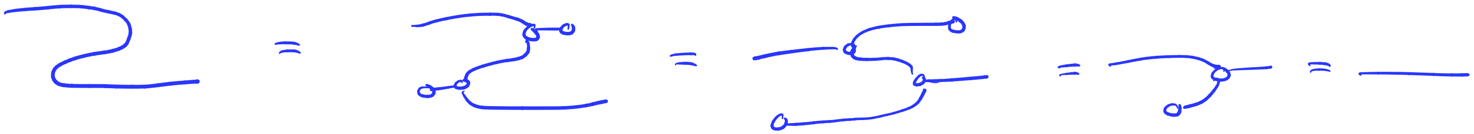
- is associative
- is unital
- is commutative
- is special
- satisfies Frobenius law

# Self-duality

set



then

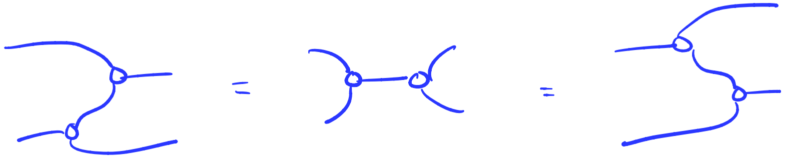


don't need decorating arrows; bits instead of qubits.

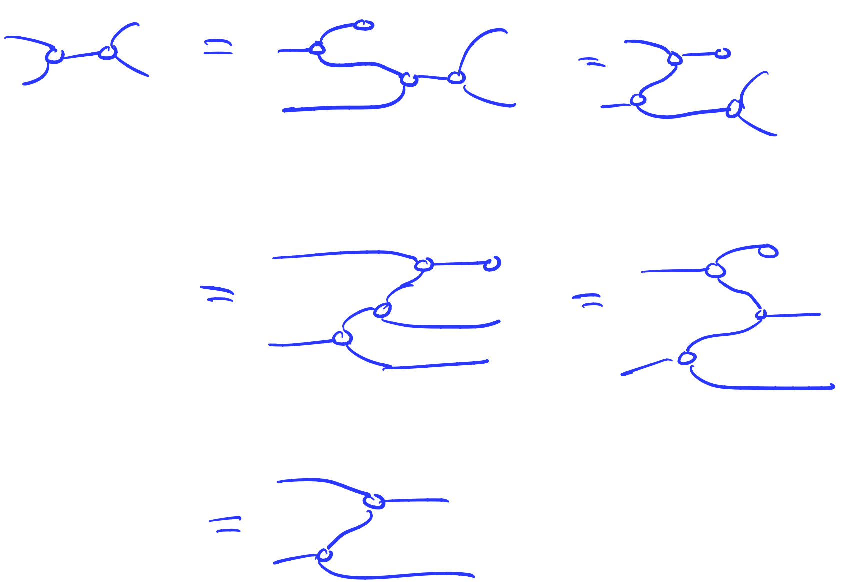
# Spiders

# Normal form

e.g.:



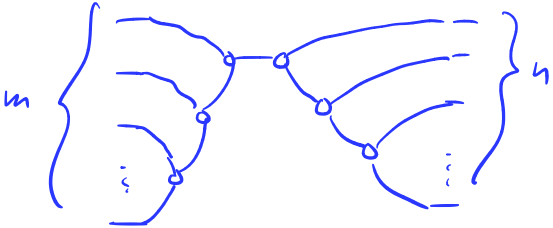
Pf:



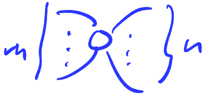
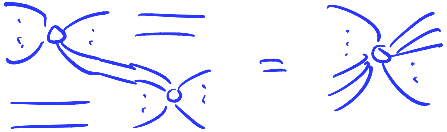
# Spiders

theorem: any connected diagram built up from 

is equal to:

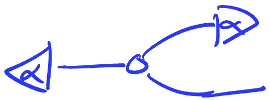



So: instead of copying maps can have <u>spider</u>

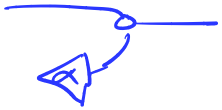
i.e. a family  of maps s.t. 

# Phases

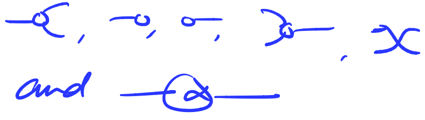
# Phases

are states  s.t.  = 

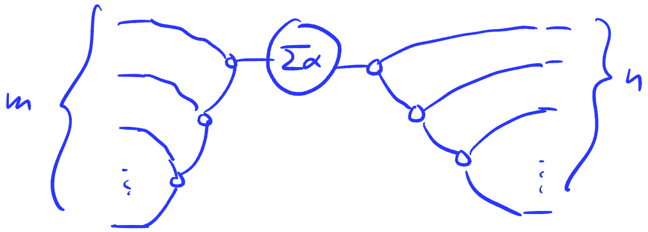
e.g.: if  copies $\{|i\rangle\}$, then phases are vectors $\alpha = \sum_i a_i |i\rangle$ with $\sum_i |a_i|^2 = 1$

<u>phase shift</u> is a map  := 

# Phased spider theorem

theorem: any connected diagram built up from 
and

is equal to:

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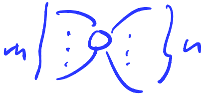
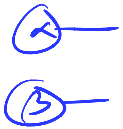
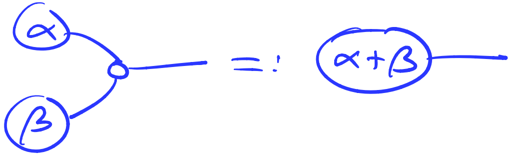
i.e. a family 

of maps s.t.

# Phase group

if 

are phases, then so is 

# Summary:

- Qiskit is fairly low-level circuit description language
- Frobenius law is extreme form of ‘only connectivity matters’
- Can equivalently think of classical data as spiders
- Spiders can carry phases around