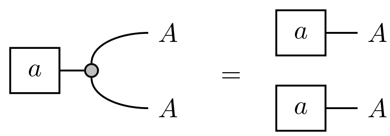
Introduction to Quantum Programming and Semantics 2025
Tutorial week 4

Exercise 1

(a) The copy map $d\colon I \longrightarrow I \otimes I$ is the empty picture with (once) zero wires in and (twice) zero wires out. Similarly, the identity map $I \longrightarrow I$ is the empty picture with (once) zero wires in and (once) zero wires out. Hence the associativity condition holds vacuously: the left-hand side is a composition of empty pictures, as is the right-hand side. Commutativity similarly holds vacuously, because the swap map $I \otimes I \longrightarrow I \otimes I$ is also the empty picture with (twice) zero wires in and (twice) zero wires out. Finally, as discarding map $e\colon I \longrightarrow I$ we can also take the empty picture, making the unitality condition true in the same way.

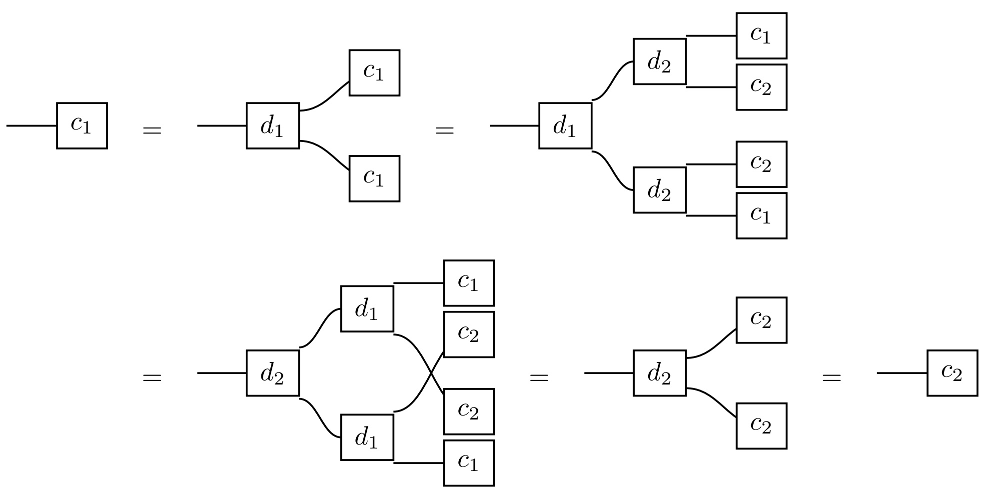
(b) Specialising the definition in the question, a homomorphism $a\colon I \longrightarrow A$ is a map satisfying:



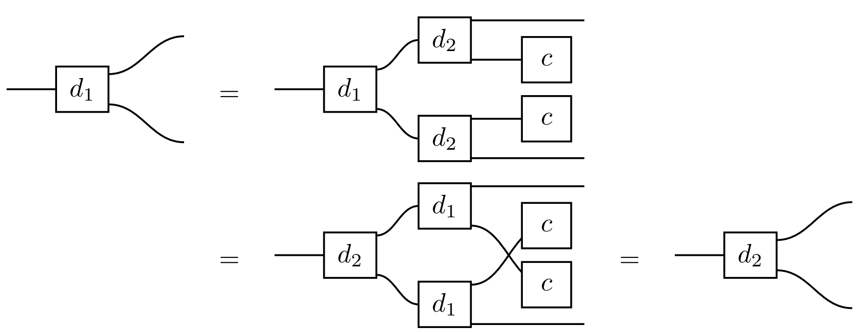
But that is exactly $f \circ a = a \otimes a$.

Exercise 2

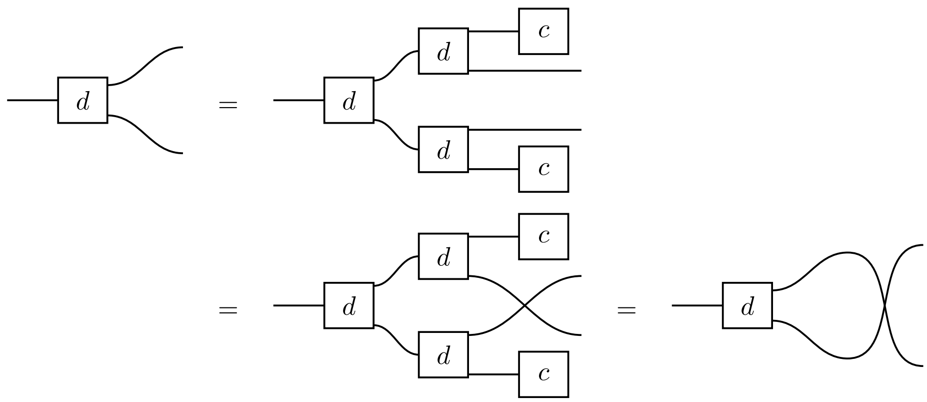
(a)



(b) Writing c for $c_1 = c_2$:



(c) Writing d for $d_1 = d_2$:

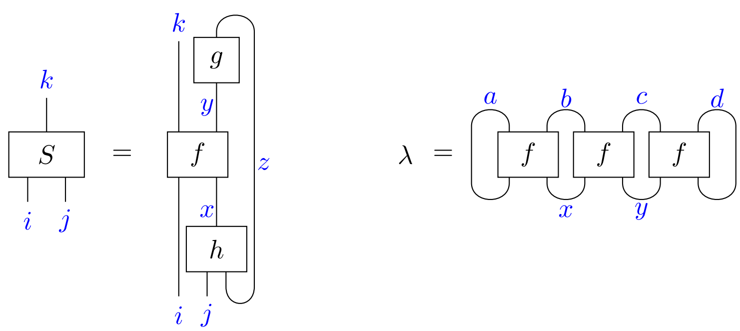


Exercise 3

$$\Phi = \mathrm{Tr}((f \otimes g) \circ (\mathrm{SWAP} \otimes \mathrm{id}) \circ (\mathrm{id} \otimes g))$$

Exercise 4

Labelling the wires with some index names:



...we get:

$$S_{ij}^{k} = \sum_{xyz} f_{ix}^{ky} g_{y}^{z} h_{jz}^{x}$$

$$\lambda = \sum_{abcdxy} f_{ax}^{ab} f_{xy}^{bc} f_{yd}^{cd}$$

Note λ is a scalar, so all indices on the RHS are summed over.