

Introduction to Quantum Programming and Semantics 2025

Tutorial week 4

Exercise 1

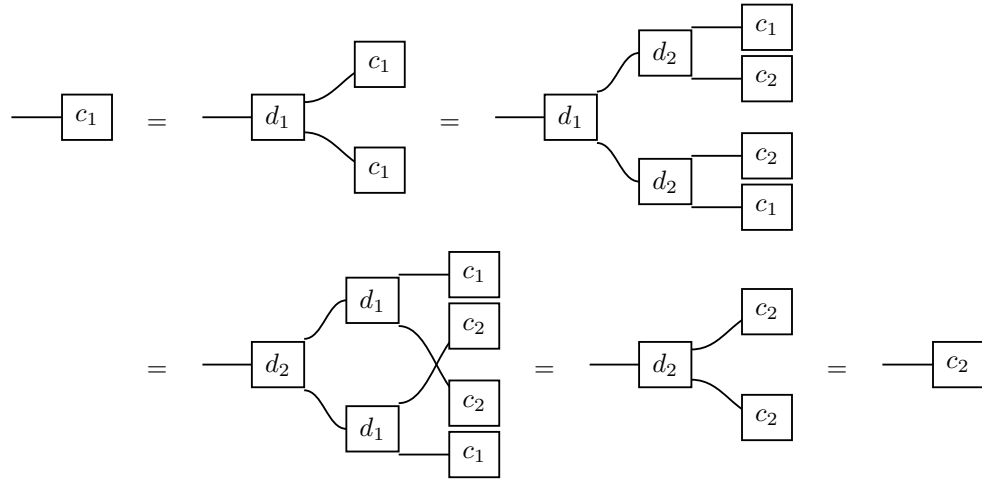
- (a) The copy map $d: I \rightarrow I \otimes I$ is the empty picture with (once) zero wires in and (twice) zero wires out. Similarly, the identity map $I \rightarrow I$ is the empty picture with (once) zero wires in and (once) zero wires out. Hence the associativity condition holds vacuously: the left-hand side is a composition of empty pictures, as is the right-hand side. Commutativity similarly holds vacuously, because the swap map $I \otimes I \rightarrow I \otimes I$ is also the empty picture with (twice) zero wires in and (twice) zero wires out. Finally, as discarding map $e: I \rightarrow I$ we can also take the empty picture, making the unitality condition true in the same way.
- (b) Specialising the definition in the question, a homomorphism $a: I \rightarrow A$ is a map satisfying:

$$\begin{array}{c} \text{---} a \text{---} \bullet \text{---} \begin{array}{l} \text{---} A \\ \text{---} A \end{array} \end{array} = \begin{array}{c} \begin{array}{c} \text{---} a \text{---} A \\ \text{---} a \text{---} A \end{array} \end{array}$$

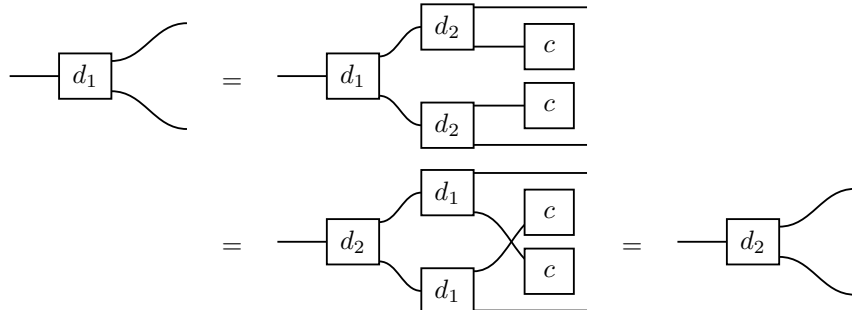
But that is exactly $f \circ a = a \otimes a$.

Exercise 2

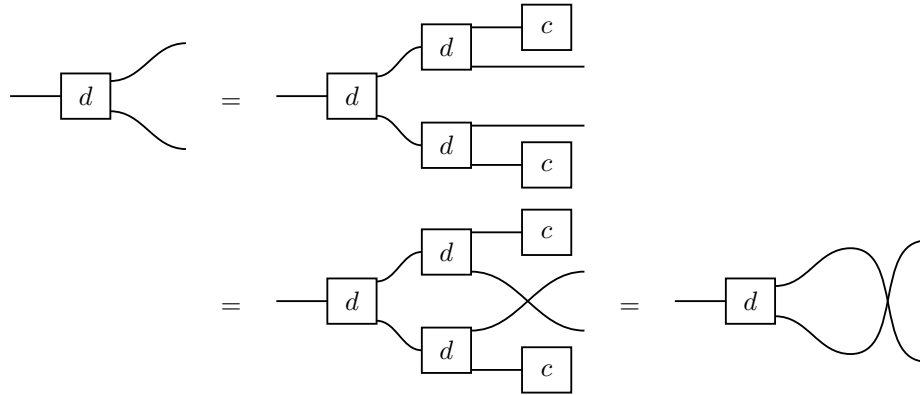
- (a)



- (b) Writing c for $c_1 = c_2$:



(c) Writing d for $d_1 = d_2$:

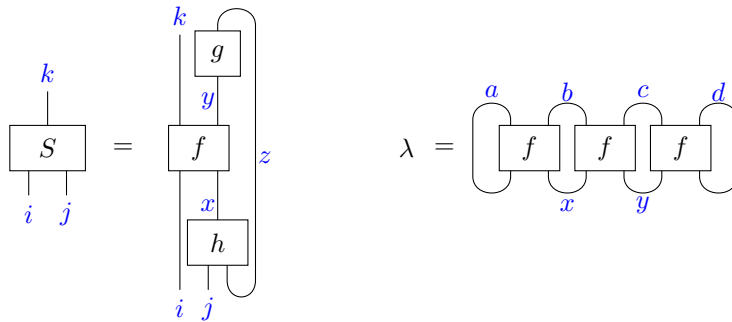


Exercise 3

$$\Phi = \text{Tr}((\text{id} \otimes g) \circ (\text{SWAP} \otimes \text{id}) \circ (f \otimes h))$$

Exercise 4

Labelling the wires with some index names:



...we get:

$$S_{ij}^k = \sum_{xyz} f_{ix}^{ky} g_y^z h_{jz}^x$$

$$\lambda = \sum_{abcdxy} f_{ax}^{ab} f_{xy}^{bc} f_{yd}^{cd}$$

Note λ is a scalar, so all indices on the RHS are summed over.