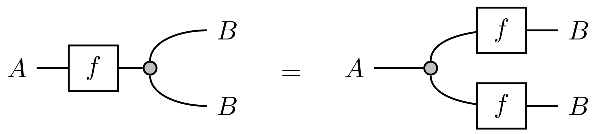
Introduction to Quantum Programming and Semantics 2025
Tutorial week 4

Exercise 1

If systems A and B have copying maps, we say that $f\colon A \longrightarrow B$ is a homomorphism when:

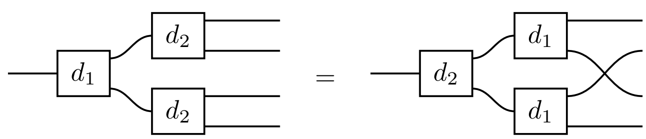


(a) Show that the empty system I (that pictorially consists of zero wires) always has a copy map that is associative, commutative, and unital.

(b) Show that a homomorphism $a\colon I \longrightarrow A$ is the same thing as a state of A that is copyable in the sense that it satisfies $f \circ a = a \otimes a$.

Exercise 2

Suppose you have two copy maps $d_1, d_2\colon A \longrightarrow A \otimes A$ that are associative, commutative, and unital with $c_1, c_2\colon A \longrightarrow I$, and that interact in the following way:



(In terms of Exercise 1: each copy map is a homomorphism for the other.)

(a) Show that $c_1 = c_2$.

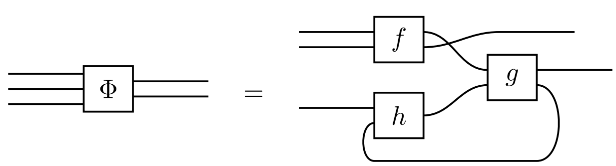
(b) Show that $d_1 = d_2$.

(c) Show that d_1 is commutative.

(This is called the *Eckmann-Hilton argument*.)

Exercise 3

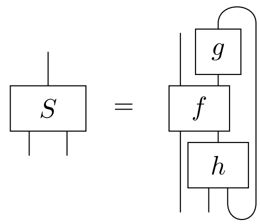
Write an algebraic expression for the following diagram using $\otimes$, $\circ$, SWAP, and $\mathrm{Tr}(-)$.



Show that it evaluates to $\Phi^{de}_{abc} = \sum_{xyz} f^{xd}_{ab} g^{ez}_{xy} h^{y}_{cz}$.

Exercise 4

Write the following diagrams as tensor contractions, i.e. as sums over products of matrix elements f^{kl}_{ij}, etc.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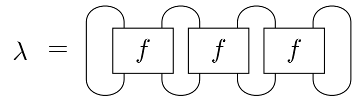
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