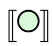
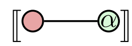
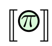
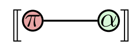
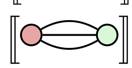
Introduction to Quantum Programming and Semantics 2025
Tutorial week 6

Exercise 1

In the notes and lectures we ignore scalar factors in ZX-diagrams, but we can represent any scalar we want with a ZX-diagram. For instance, we have:

 $= 2$  $= \sqrt{2}$

 $= 0$  $= \sqrt{2}e^{i\alpha}$ $(*)$

 $= 1 + e^{i\alpha}$  $= \frac{1}{\sqrt{2}}$

By combining the diagrams from $(*)$, find a ZX-diagram to represent the following scalars z:

(a) $z = -1$.

(b) $z = e^{i\theta}$ for any θ.

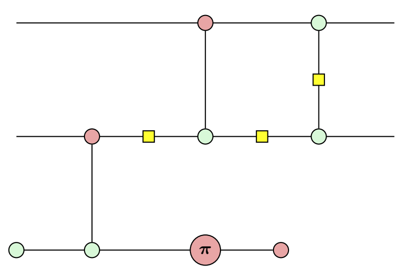
(c) $z = \frac{1}{2}$.

(d) $z = \cos\theta$ for any value θ.

(e) Describe a systematic way to construct the ZX-diagram for any complex number z.

Exercise 2

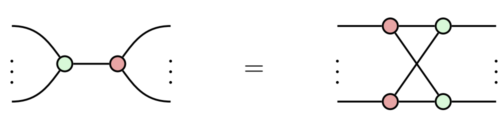
Using ZX-calculus rewrites, help the poor trapped π phase find it's way to an exit (i.e. an output).



Note that it might be leaving with friends.

Exercise 3

Prove the following rule by induction on the number of input and output wires:



Here, the ZX diagram on the right is the full connected bipartite graph.

Exercise 4

Prove the following quantum circuit identity by translating to ZX diagrams and rewriting: