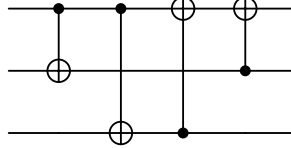


Introduction to Quantum Programming and Semantics 2025

Tutorial week 7

Exercise 1

Consider the following CNOT circuit:



- (a) Calculate the parity matrix of this circuit.
- (b) Resynthesise a new, equivalent, CNOT circuit from this parity matrix by using Gauss-Jordan reduction (as in [KW Theorem 4.1.7]).

Exercise 2

Consider the following parity matrix:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

What goes wrong if you try to rewrite it into a CNOT circuit (as in [KW Proposition 4.2.12])?

Exercise 3

Show that a ZX diagram in parity normal form is unitary if and only if its biadjacency matrix is invertible.

Exercise 4

Verify that the following OpenQASM programs are NOT semantically equivalent:

```

qubit a;                                     result = 1;
qubit b;
reset b;
reset a;
H a;
X b;
H b;
CX a, b;
sy a;
result = measure a;

```

Here, the gate `sy` is interpreted as the matrix $\begin{pmatrix} \frac{1+i}{2} & \frac{-1-i}{2} \\ \frac{1-i}{2} & \frac{1+i}{2} \end{pmatrix}$.

How might knowing that the state of a qubit is always $|0\rangle$ or $|1\rangle$ help a compiler writer?