Introduction to Quantum Programming and Semantics 2025
Tutorial week 8

Exercise 1

Compute

$$|H'\rangle = (e^{i\frac{\pi}{8}} + e^{-i\frac{\pi}{8}})\,|0\rangle - i(e^{i\frac{\pi}{8}} - e^{-i\frac{\pi}{8}})\,|1\rangle\,.$$

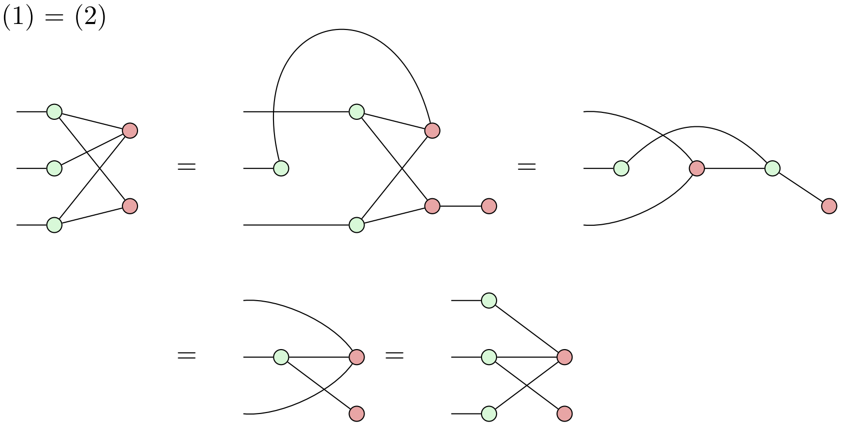
It then follows that

$$e^{-i\frac{\pi}{8}} HS\,|H'\rangle = \sqrt{2}\,|0\rangle + \sqrt{2}e^{-i\frac{\pi}{4}}\,|1\rangle\,.$$

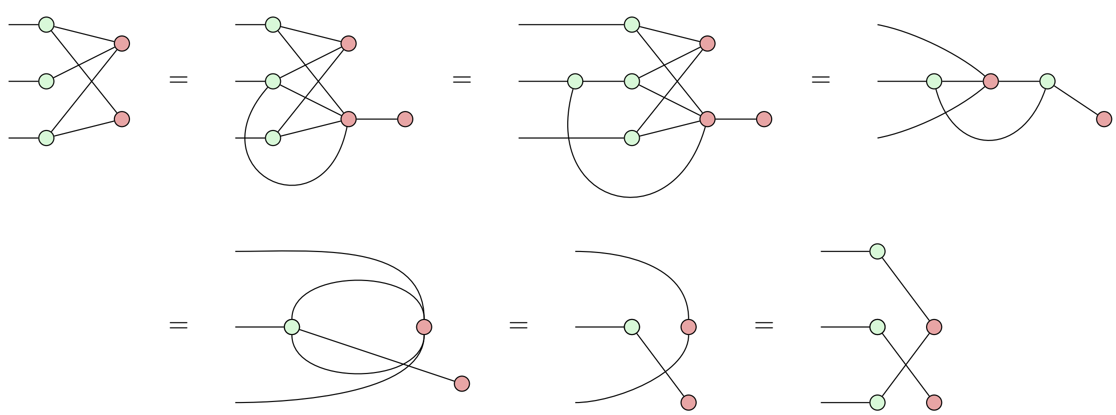
Hence $S^\dagger H T^\dagger \,|+\rangle = |H\rangle$ up to global phase.

Exercise 2

(1) = (2)



(1) = (3)



Exercise 3

Composing the normal form with S again gives a normal form by spider fusion. When we compose a H we make a case distinction on b. If $b = 0$, then a and c fuse, and we can just decompose the H into its ZXZ Euler decomposition to get a normal form again after spider fusion. We can do something similar when $b = 2$, by moving the X gate out of the way first. If $b = 1$, we realise the normal form can also be written as $Z((c-1)\frac{\pi}{2})HZ((a-1)\frac{\pi}{2})$ by using the Euler decomposition in reverse. The result is then easily proven. The same when $b = 3$.

Exercise 4

The circuits are:

(a) $X = HZH = HSSH$

(b) HX

(c) $CX(H \otimes 1)$

(d) $(1 \otimes X)CX(H \otimes 1)$

(e) $CX_{1,3}CX_{1,2}(H \otimes 1 \otimes 1)$