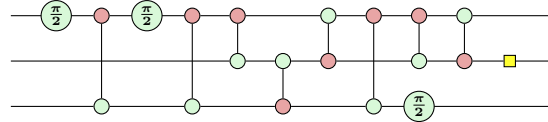


Introduction to Quantum Programming and Semantics 2025

Tutorial week 9

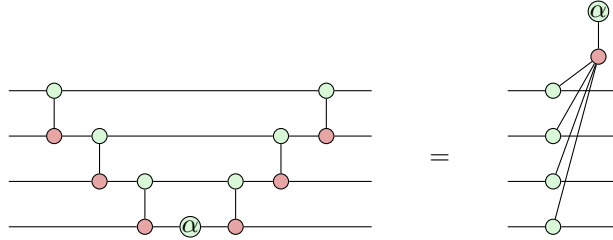
Exercise 1

Simplify the following circuit to a diagram that has no internal spiders with a $\pm\frac{\pi}{2}$ phase or pairs of internal spiders with a 0 or a π phase.



Exercise 2

Show that phase gadgets have the following decomposition:



(Hint: Use equations (7.13), (7.14), and (7.15) from [KW].)

Exercise 3

Define an *H-box* as follows:

$$m \left\{ \begin{array}{c} \vdots \\ \vdots \\ \text{[yellow box with } a \text{]} \\ \vdots \\ \vdots \end{array} \right\} n = \sum a^{i_1 \dots i_m j_1 \dots j_n} |j_1 \dots j_n\rangle \langle i_1 \dots i_m|$$

Verify the following equation by plugging in computational basis states and checking that it gives the correct phase:

$$CCZ = \sqrt{2} \begin{array}{c} \text{[circuit diagram with three lines and a yellow box with -1]} \end{array} \quad (*)$$

Exercise 4

Show by ZX diagram rewriting that the CCZ gate as defined in (*) is its own inverse (up to scalar normalisation).