Introduction to Quantum Programming and Semantics

Lecture 12: Measurement

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Overview

- Q#
- Measurement
- Mixed states
- Quantum channels
- Environment structures

Q#

Microsoft QDK

- Q# language
- Q# libraries with several standard operations and algorithms
- Dot.net integration with classical languages (Python, C#, F#, etc)
- Orchestration language to execute Q#:
    - Simulators
    - Resource estimators
    - Microsoft Azure quantum hardware

Q#

- Language itself is imperative …

- … but can be used in functional way through dot.net

- Not (necessarily) circuit description language:
  quantum instructions are dispatched in order
  and you can use measurement results in rest of program

  (cf measurement-based quantum computing, quantum error correction)

Measurement

Measurement is the only way to get info out of quantum system

for state $|\psi\rangle$, will tell you: - probability of outcome j

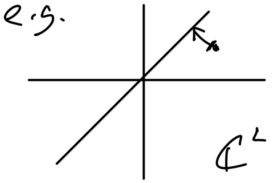
- post-measurement state

a <u>projection</u> is a linear map $\mathbb{C}^n \xrightarrow{P} \mathbb{C}^n$ s.t. $P^\dagger = P^2 = P$ (self-adjoint; idempotent)

splits $\mathbb{C}^n$ into two pieces: $-\ \mathrm{im}(P) = \{|\psi\rangle \mid P|\psi\rangle = |\psi\rangle\}$

$-\ \mathrm{im}(P)^\perp = \{|\psi\rangle \mid P|\psi\rangle = 0\} = \mathrm{im}(1-P)$

e.g.



$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

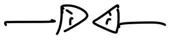
Measurement

more generally, $P_1, \dots, P_k$, where $\sum_{j=1}^{k} P_j$ split into k orthogonal pieces.

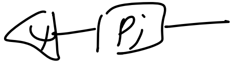
<u>von Neumann measurement</u> $\mathcal{M} = \{P_1, \dots, P_k\}$ s.t. $\sum_{j=1}^{k} P_j = 1$

probability of outcome j when in state $|\psi\rangle$ is $\langle\psi|P_j|\psi\rangle$ ← "Born rule"

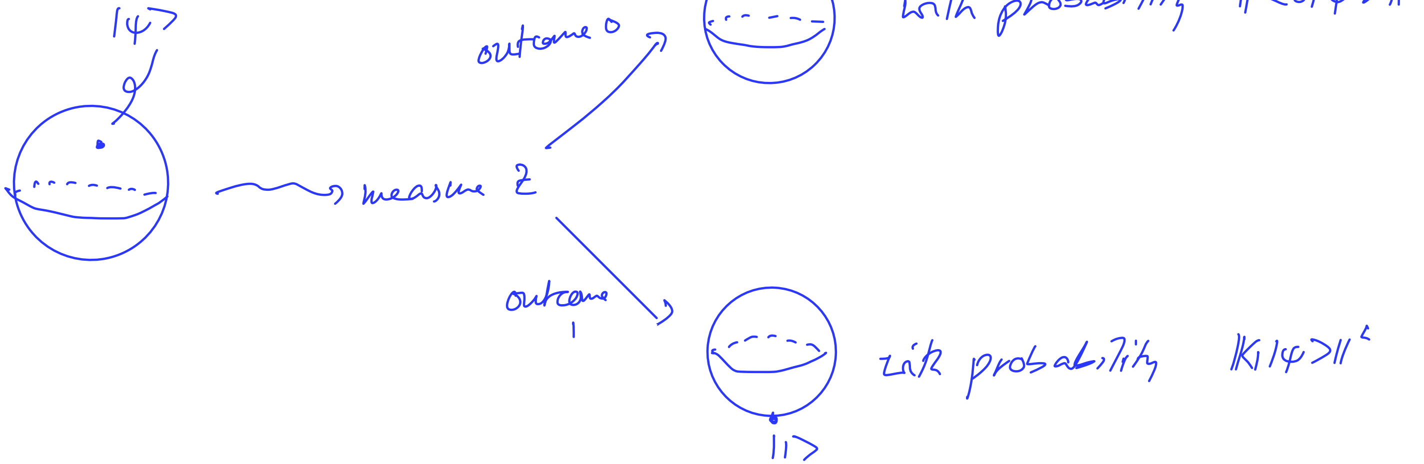
note: if $|\psi\rangle = e^{i\theta}|\varphi\rangle$ then $\langle\psi|P_j|\psi\rangle = \langle\varphi|P_j|\varphi\rangle$

if {  } orthonormal basis, then {  } is a vN-measurement

Measurement

after measurement: $|\psi\rangle \overset{\text{project}}{\rightsquigarrow}$  $\overset{\text{normalise}}{\rightsquigarrow} \frac{1}{\| P_j |\psi\rangle \|} \cdot$ 

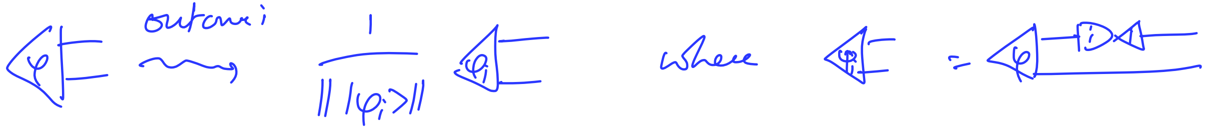
e.g. $Z = \{ |0\rangle\langle 0|, |1\rangle\langle 1| \}$



Mixed states

Mixed states

measure subsystem only : $M = \{$  $\}$



if qubits not entangled, $\varphi = \psi_1 \otimes \psi_2$, then get $|i\rangle \otimes |\psi_2\rangle$ → pure state

if qubits entangled, post-measurement $\sum_i |i\rangle \otimes |\psi_i\rangle$ → mixed state = convex combination of pure states

density matrix = positive semidefinite, trace 1

$= \sum_{i=1}^{n} p_i |i\rangle\langle i|$ for some probability distribution p_i on orthonormal basis $\{|i\rangle\}$

Quantum channels

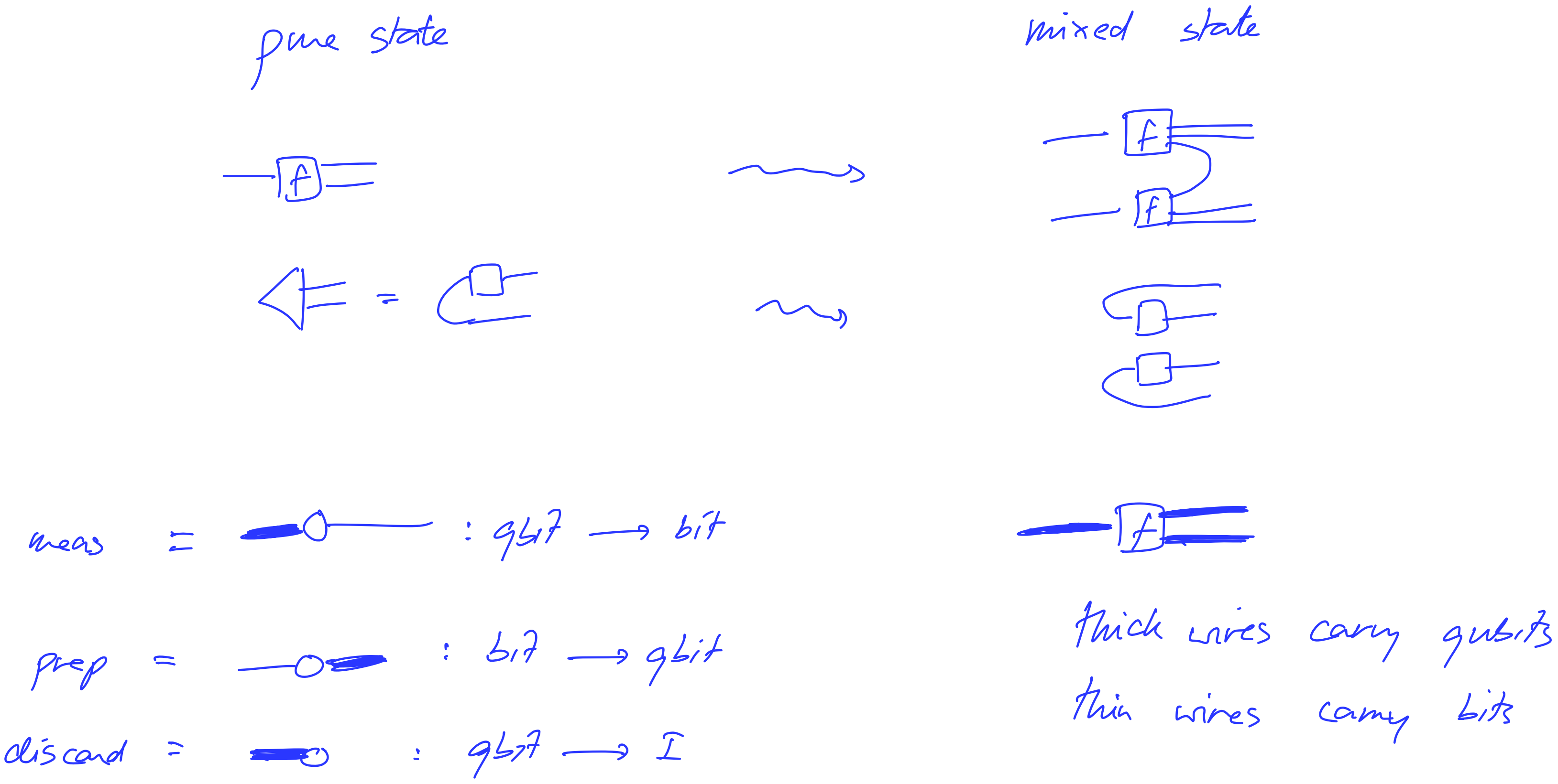
Completely positive maps

$$\mathbb{C}^{m^2} \xrightarrow{f} \mathbb{C}^{n^2}$$

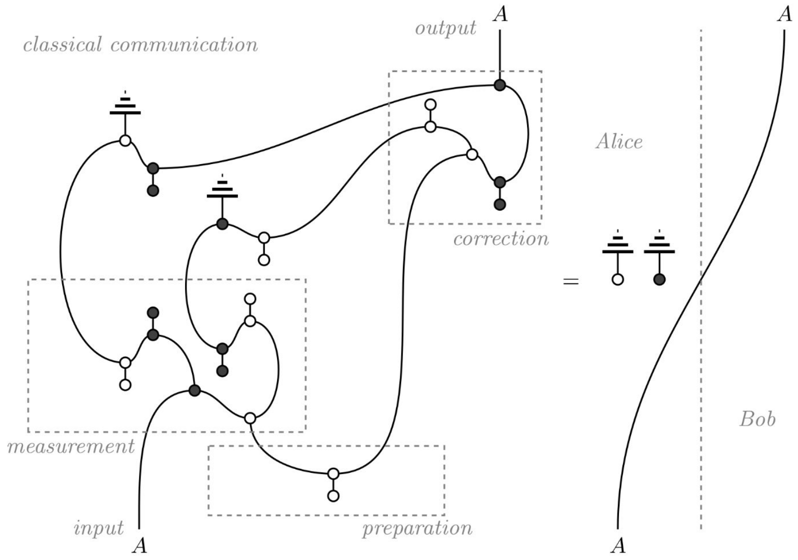
$$f(\rho) = \sum_i g_i^\dagger \rho g_i$$

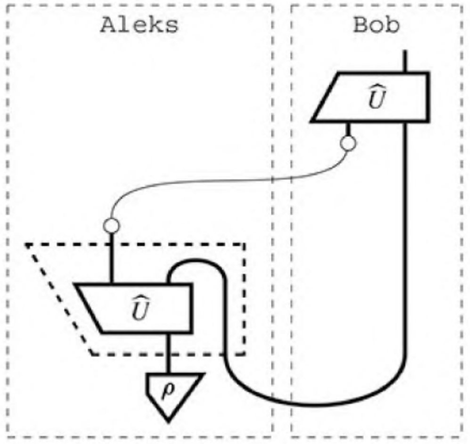
Environment structures

Thick vs thin wires



Quantum teleportation





Summary:

- Q#: not necessarily circuit description, mid-circuit measurement
- Mixed states: partial knowledge
- Measurement: postselection, sums, graphically