

Introduction to Quantum Programming and Semantics

Lecture 15: Quantum simulation

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Overview

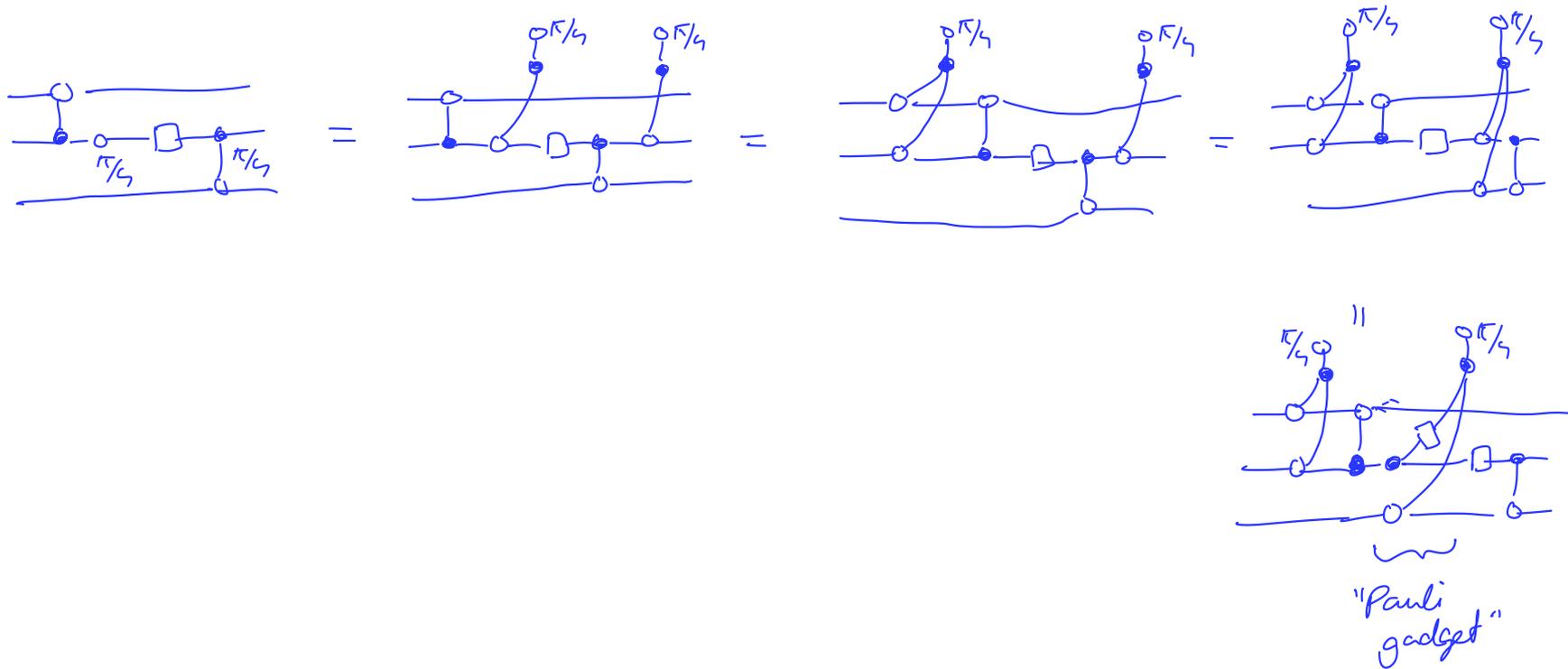
- Pauli gadgets
- Hamiltonian simulation

Pauli gadgets

Pauli gadgets

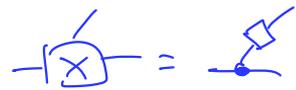
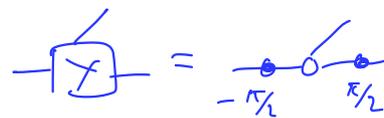
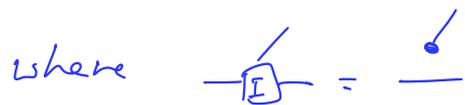
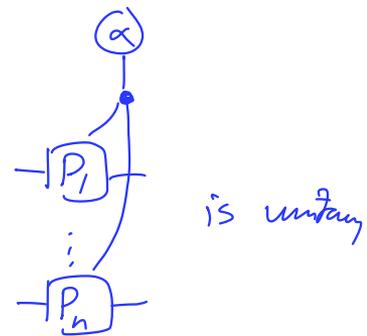
Clifford + phase is universal gate set

Q: can we move all non-Clifford gates out?

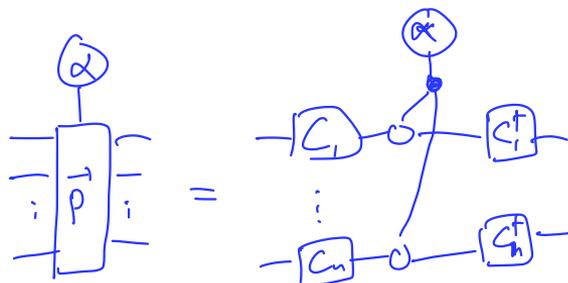


Pauli gadgets

If $P_i \in \{I, X, Y, Z\}$ and $\vec{P} = P_1 \otimes \dots \otimes P_n$, then the map

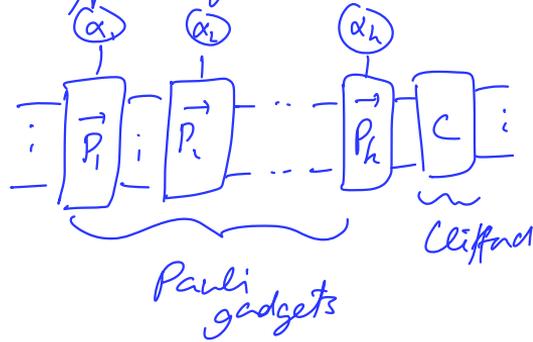


Note:



Pauli gadgets

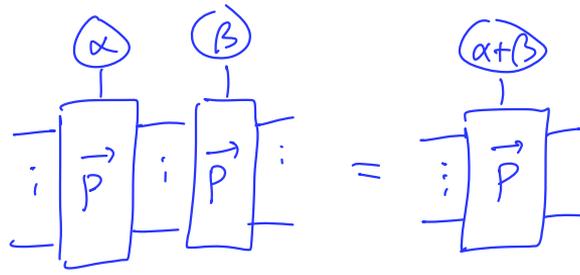
Thm: any Clifford+phase circuit can be written as



- Pf:
- Clifford gates commute past Pauli gadgets
 - can move phase out of C

Pauli gadgets

can fuse:



and if $\vec{P}\vec{Q} = \vec{Q}\vec{P}$ then $\vec{P}(\alpha)\vec{Q}(\beta) = \vec{Q}(\beta)\vec{P}(\alpha)$

- algorithm:
1. compute Pauli gadgets form of circuit
 2. commute and combine Pauli gadgets phases whenever possible
 3. merge Pauli gadgets with Clifford phases into Clifford part
 4. repeat
 5. extract circuit (many options)

Hamiltonian simulation

self adjoint
matrix

Hamiltonian simulation

$$|\psi(t)\rangle = e^{-itH} |\psi_0\rangle$$

Q: design circuit implementing e^{-itH}

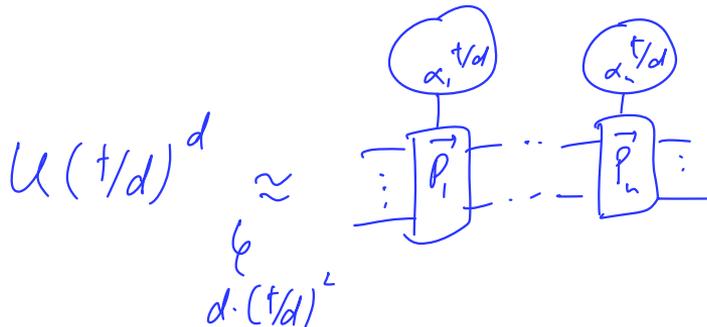
observation: Pauli span space of all self-adjoint matrices: $H = \sum_j P_j \alpha_j$

if all P_j commute, then easy $U = e^{-itH} = e^{-it\alpha_1 P_1} \dots e^{-it\alpha_n P_n}$

if don't commute, not true, but can still approximate

trick ("Trotterisation") make t very small

$$U = e^{-itH} = (e^{-itH/d})^d$$



error $d (t/d)^2$
 $t/d \rightarrow 0$ as $d \rightarrow \infty$

Summary:

- ZX diagrams with phases $\pi/4$ are fully universal but difficult
- Pauli gadgets can graphically handle exponentials of Pauli matrices
- Can simulate Hamiltonians by chopping into small time steps