Introduction to Quantum Programming and Semantics

Lecture 16: Oracles

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Overview

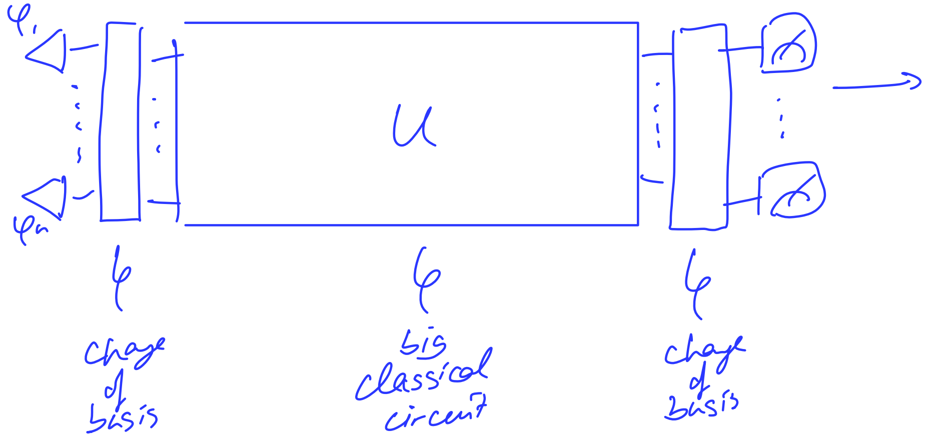
- Quipper
- Oracles

Types of quantum algorithms

main trick: change of basis

$-\boxed{H}-$ or more generally quantum Fourier transform

① "oracular" quantum algorithms



- Deutsch-Jozsa, Simon's
- Factoring (Shor): reduce factoring to period-finding

  $U(a) = a \bmod n + q$

  find n

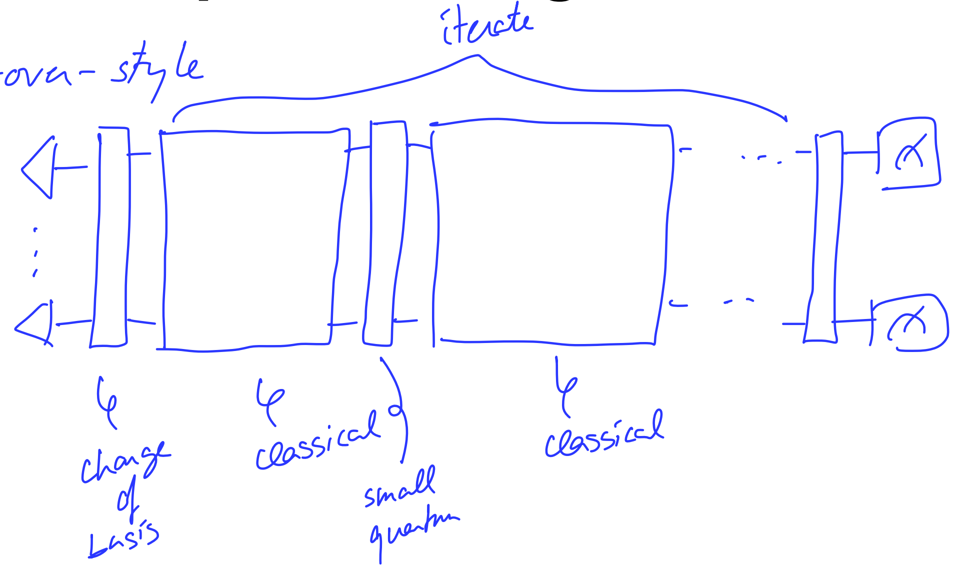
- Hidden subgroup

  $i : G \to H$ injection of abelian groups

  find G

Types of quantum algorithms

② Grover-style



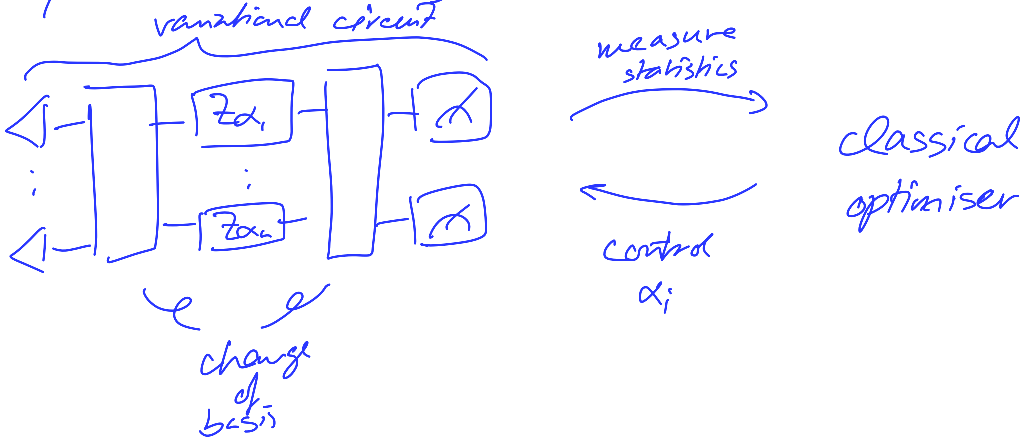
- Grover search
- amplitude amplification
- quantum walks

③ Hamiltonian simulation

- Trotterisation
- linear combination of unitaries
- QSP : quantum signal processing
  QSVT : quantum singular value transform

Types of quantum algorithms

④ Hybrid quantum-classical algorithms / quantum machine learning



Quipper

Quipper

- Open source
- Functional (with side effects)
- Domain-specific language in Haskell
- Aim: resource estimation
- Lazy
- Library including 7 nontrivial reference quantum algorithms

Quipper model

Execution phases:

- Compile time

- Circuit generation time
  Inputs whose values are already known now are called *parameters*
  e.g. Deutsch-Jozsa is really a family of circuits, one for each n

- Circuit execution time
  Inputs whose values are only known now are called *inputs*
  e.g. the n input qubits to Deutsch-Jozsa

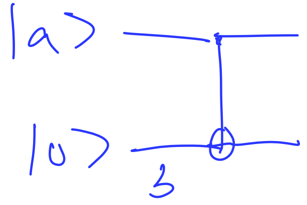
Quipper model

Types:

- Bits
    - have type Bool at circuit generation time
    - have type Bit as classical Boolean input to a circuit
- Qubits
    - Have type Qubit, only available as inputs at circuit execution time
- Bools can be converted into Bit, but not the other way around
- Measurements only at circuit execution time, so outcome is Bit, not Bool

Control

```
share :: Qubit -> Circ (Qubit, Qubit)
share a = do
  b <- qinit False
  b <- qnot b 'controlled' a
  return (a,b)
```



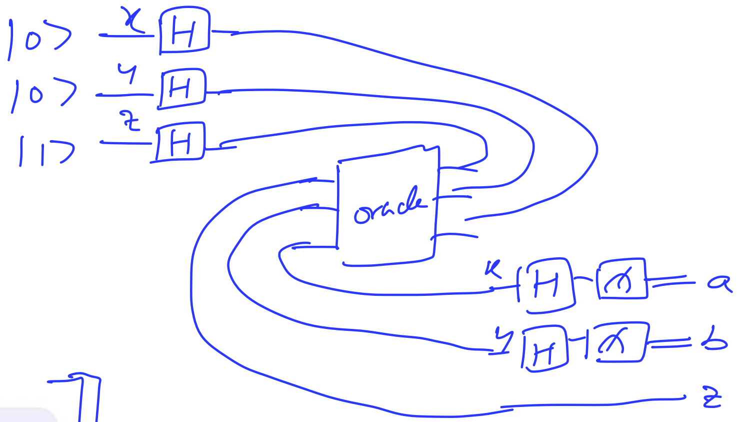
$|0\rangle \longleftrightarrow |00\rangle$

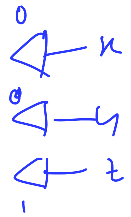
$|1\rangle \longleftrightarrow |11\rangle$

Measurement

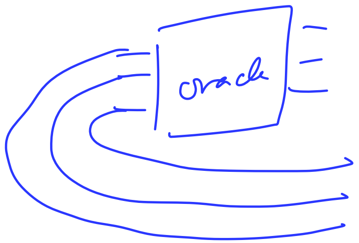
```
measurement :: Qubit -> Circ Bit
measurement q = do
  x <- measure q
  return x
```

Quipper semantics





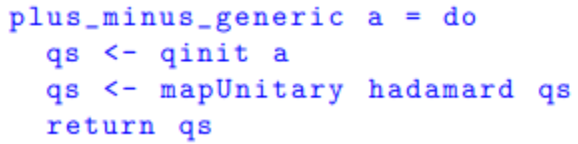
```
deutschjozsa :: (Qubit -> Qubit -> Qubit -> Circ (Qubit, Qubit, Qubit))
                -> Circ (Bit, Bit)
deutschjozsa (oracle) = do
  x <- qinit False
  y <- qinit False
  z <- qinit True
  hadamard x
  hadamard y
  hadamard z
  (x,y,z) <- oracle x y z
  hadamard x
  hadamard y
  (a,b) <- measure (x,y)
  return (a,b)
```



Advanced Quipper

- Quantum data types:
  - E.g. (Qubit,[Qubit]), (Bit,[Bit]), (Bool,[Bool])

- Generic functions:
  - Following does not just apply to Bool, but also e.g. (Bool,[Bool])



```
plus_minus_generic a = do
  qs <- qinit a
  qs <- mapUnitary hadamard qs
  return qs
```

- Recursion:
  - Circuit-producing functions can be recursive over any parameters known at circuit generation time.
  - Can e.g. recurse over list of qubits to write QFT

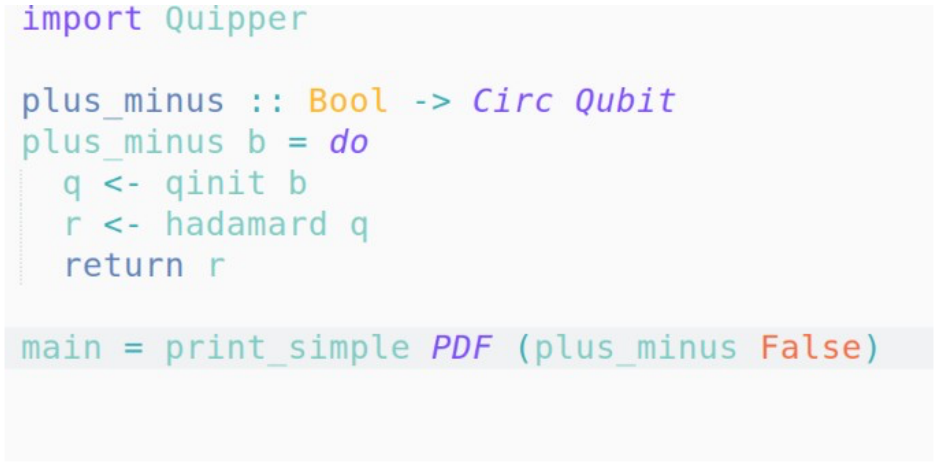
Advanced Quipper

- Circuit operations:
    - Functions that take a circuit and make new circuits based on it
    - E.g. repeat circuit number of times, reverse circuit, use as subcircuit
    - Any classical reversible function can turn into a circuit automatically:

```
build_circuit
reversiblefunction :: (Bool, Bool, Bool) -> (Bool, Bool, Bool)
reversiblefunction x y z = y z x
```

- Semantics: well-founded semantics, but not yet
    - Dependent types: recognise that Bits are only used at circuit execution
    - Linear types: prevent terms involving Qubits from being copied
      Proto-Quipper

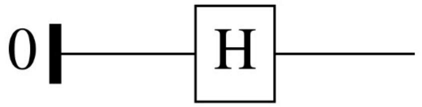
Toy example



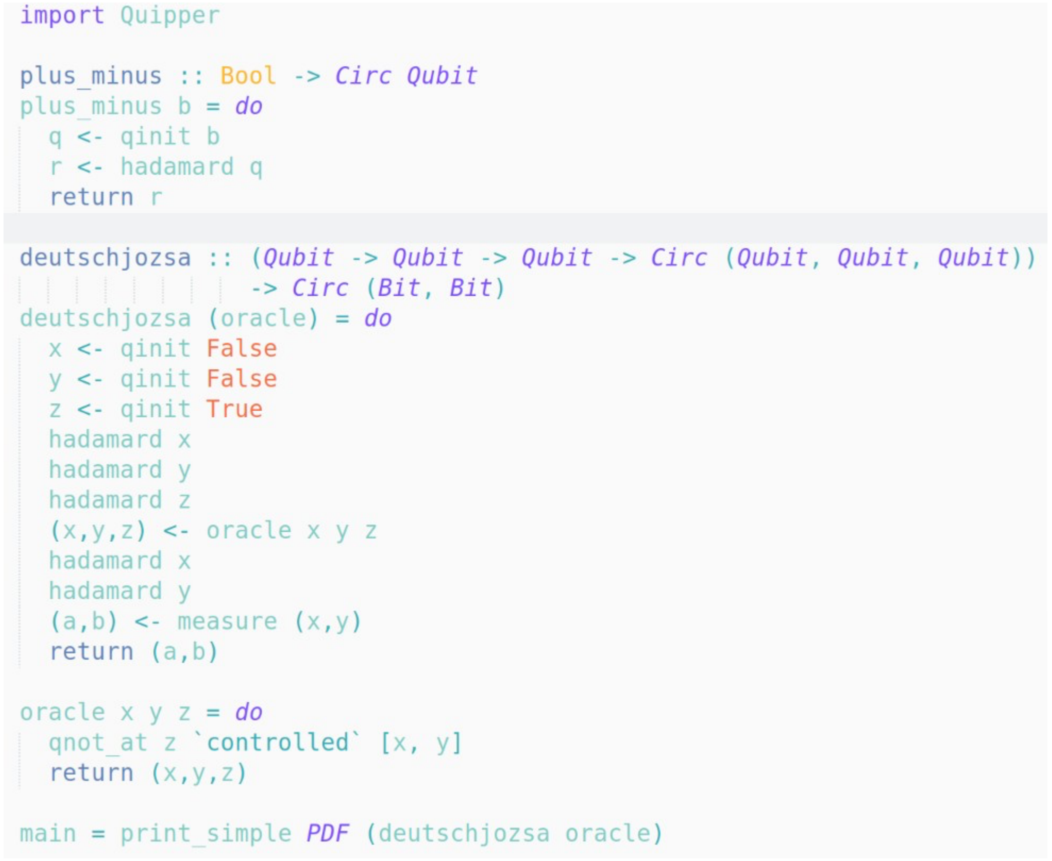
```
import Quipper

plus_minus :: Bool -> Circ Qubit
plus_minus b = do
  q <- qinit b
  r <- hadamard q
  return r

main = print_simple PDF (plus_minus False)
```



Deutsch-Josza



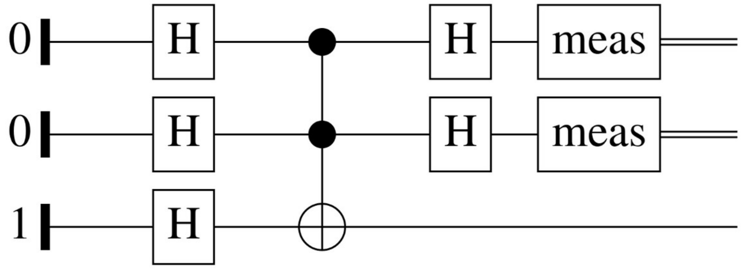
```haskell
import Quipper

plus_minus :: Bool -> Circ Qubit
plus_minus b = do
  q <- qinit b
  r <- hadamard q
  return r

deutschjozsa :: (Qubit -> Qubit -> Qubit -> Circ (Qubit, Qubit, Qubit))
                -> Circ (Bit, Bit)
deutschjozsa (oracle) = do
  x <- qinit False
  y <- qinit False
  z <- qinit True
  hadamard x
  hadamard y
  hadamard z
  (x,y,z) <- oracle x y z
  hadamard x
  hadamard y
  (a,b) <- measure (x,y)
  return (a,b)

oracle x y z = do
  qnot_at z `controlled` [x, y]
  return (x,y,z)

main = print_simple PDF (deutschjozsa oracle)
```



Summary:

- Quipper is functional, lazy, embedded
- Three phases of execution
- Higher-order so works particularly well with oracles