Introduction to Quantum Programming and Semantics

Lecture 4: Graphical calculus

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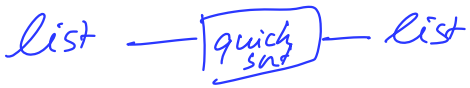
Overview

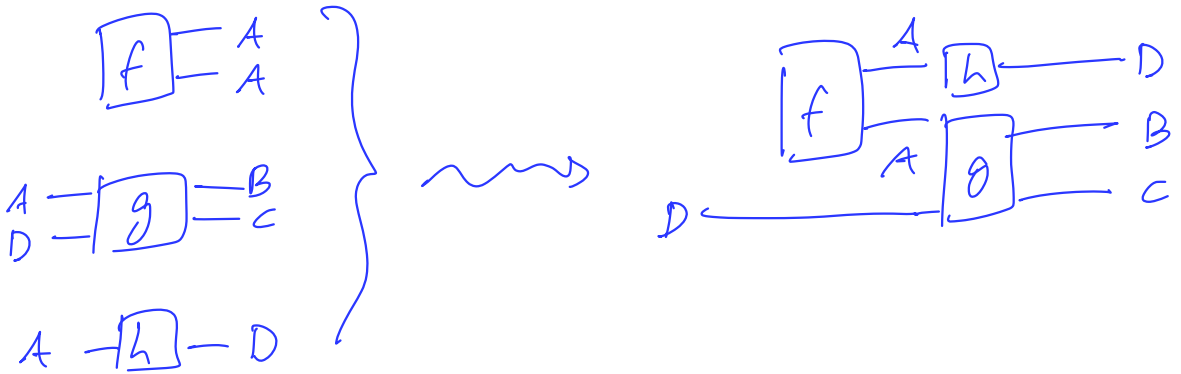
- String diagrams
- States, effects, scalars
- Entanglement

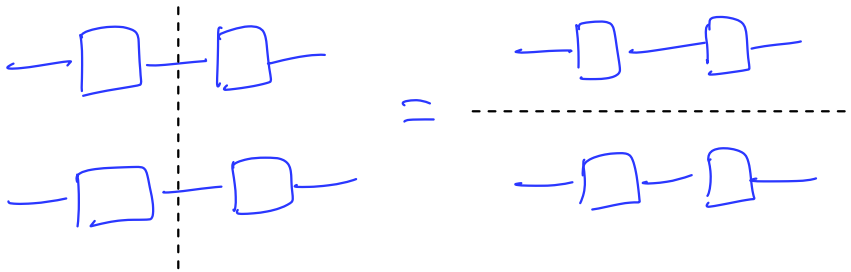
String diagrams

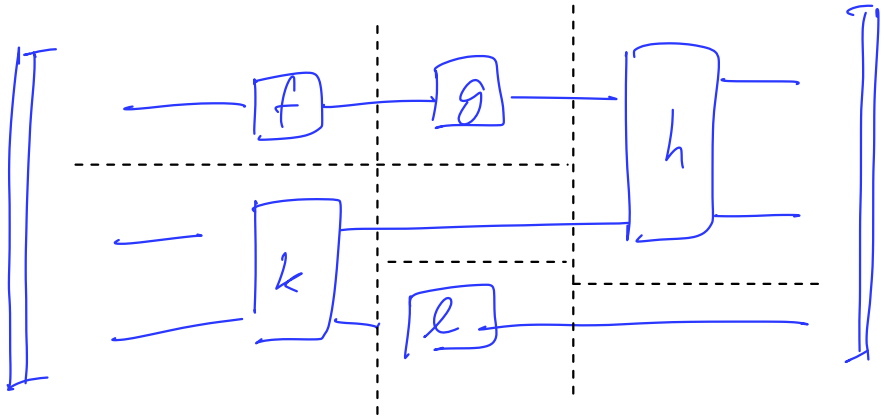
String diagrams

$f(x,y) = x^2 + y$



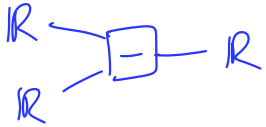


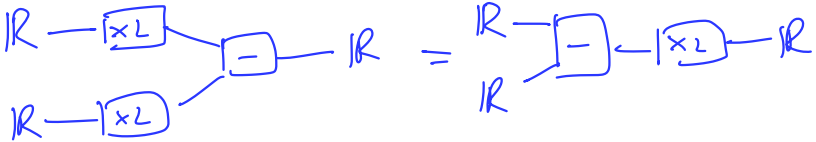


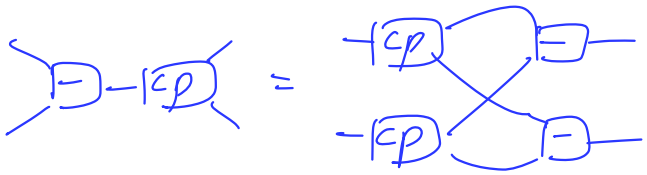


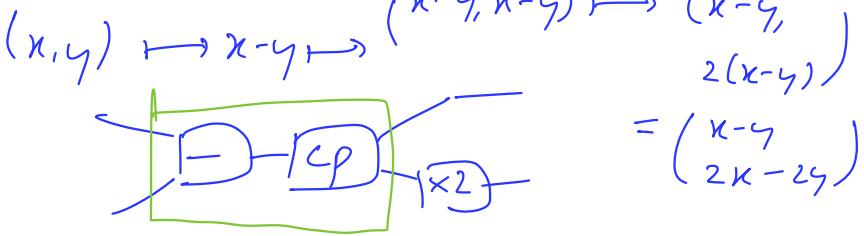
$$= (\llbracket h \rrbracket \otimes id) \circ (\llbracket g \rrbracket \otimes id \otimes \llbracket \ell \rrbracket) \circ (\llbracket f \rrbracket \otimes \llbracket k \rrbracket)$$

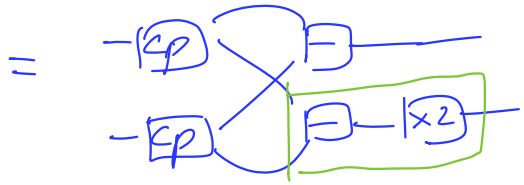
Diagrammatic reasoning

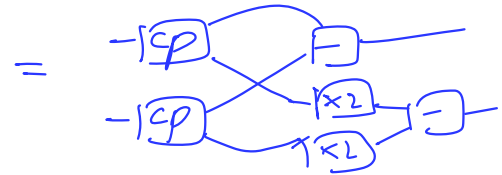


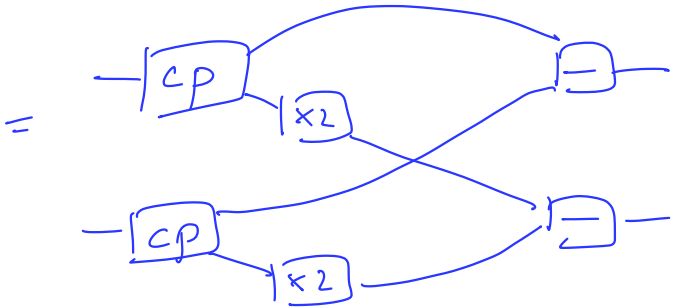




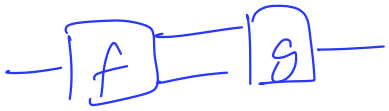


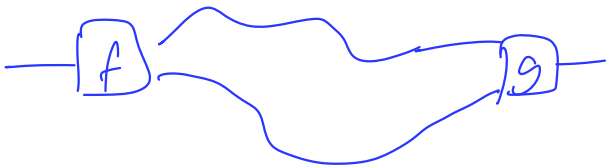






Isotopy



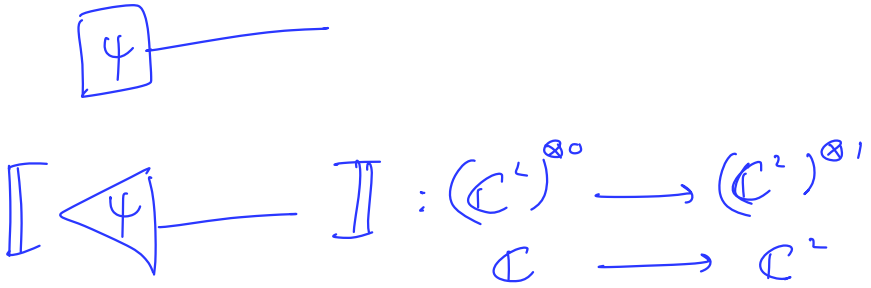


Soundness and completeness

algebra	pictures
$\otimes$, $\circ$	2d, no $\times$
$\otimes$, $\circ$, braided (topological q. comp.)	3d, $\times$
$\otimes$, $\circ$, symmetry	4d, $\times$

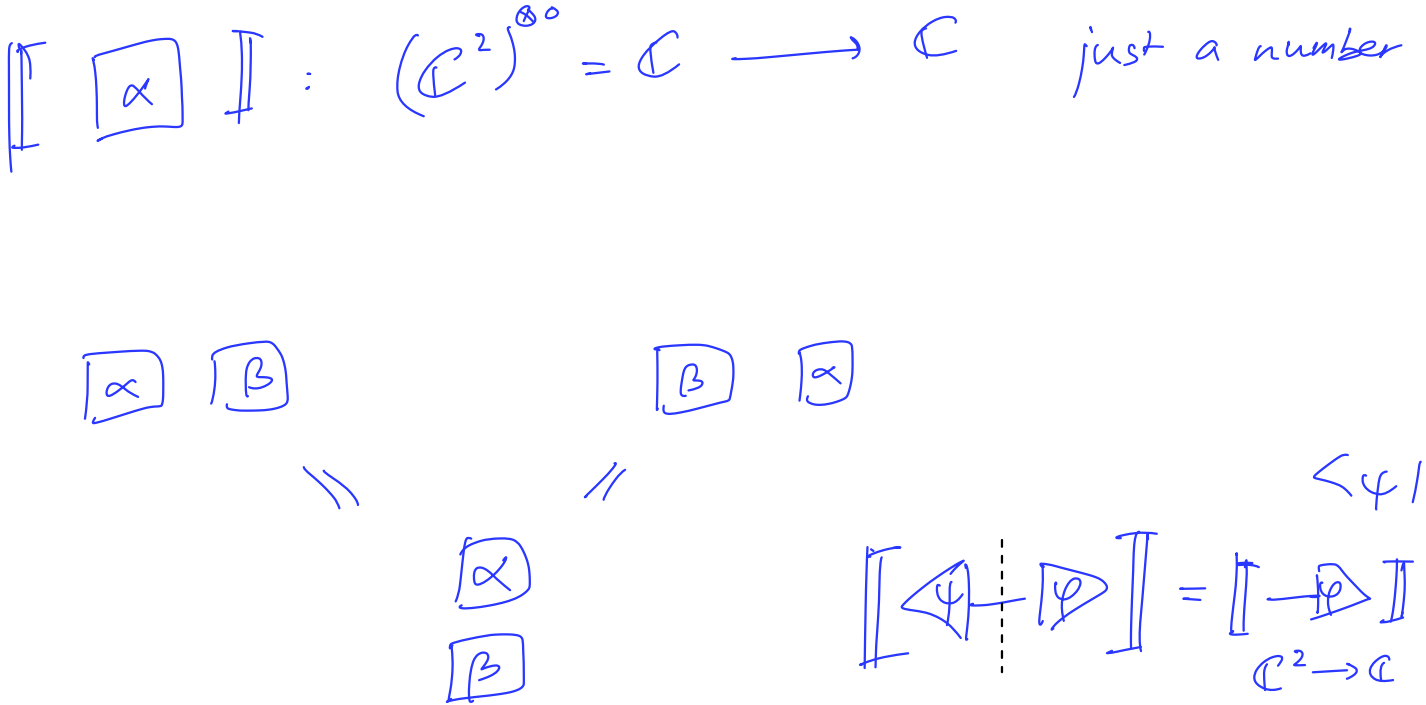
States, effects, scalars

States


ψ
ψ
$(\mathbb{C}^2)^{\otimes 0} \longrightarrow (\mathbb{C}^2)^{\otimes 1}$
$\mathbb{C} \longrightarrow \mathbb{C}^2$

Effects

Scalars


$\llbracket \alpha \rrbracket : (\mathbb{C}^2)^{\otimes 0} = \mathbb{C} \longrightarrow \mathbb{C}$
just a number
α β
β α
α
β
$\langle \psi | \varphi \rangle$
$\mathbb{C}^2 \to \mathbb{C}$
$\mathbb{C} \to \mathbb{C}^1$

Entanglement

Entanglement

bipartite state is of the form 

it is <u>separable</u> if 

for some states φ, θ

it is <u>entangled</u> otherwise

Summary:

- String diagrams have (4d) graphical calculus
- Sound and complete for sequential and parallel composition
- States, effect, and scalars are derived concepts
- Entanglement makes sense at graphical level