

Introduction to Quantum Programming and Semantics

Lecture 4: Graphical calculus

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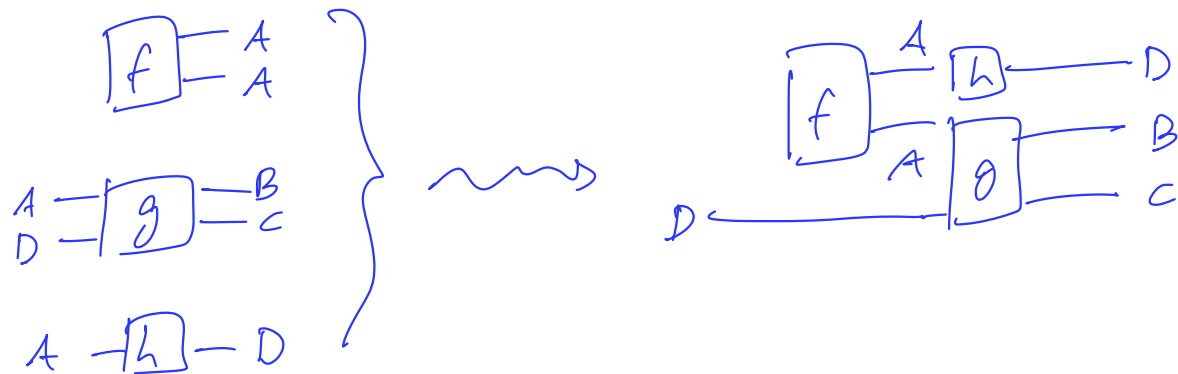
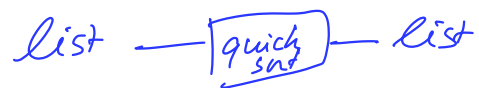
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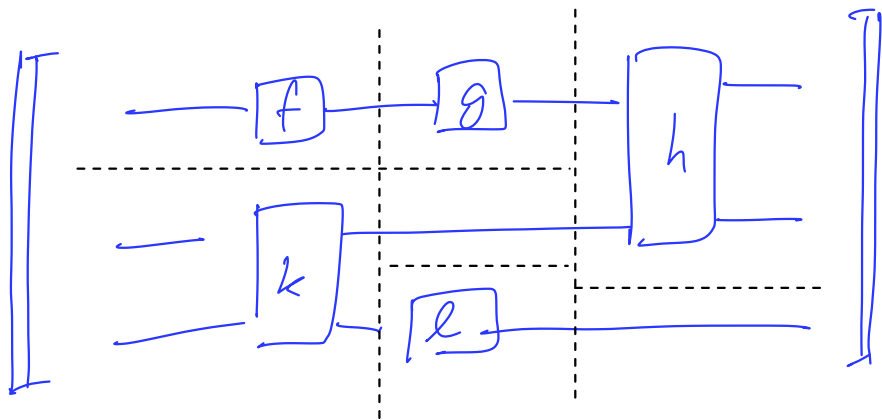
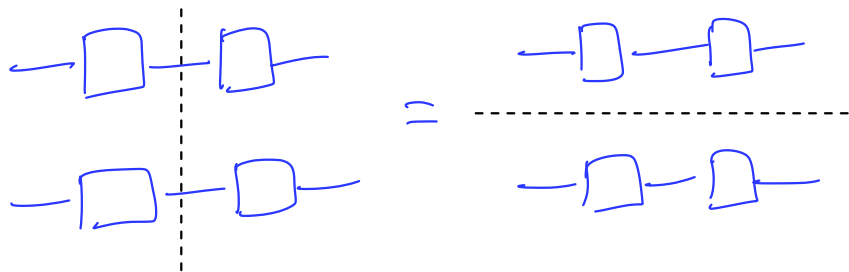
Overview

- String diagrams
- States, effects, scalars
- Entanglement

String diagrams

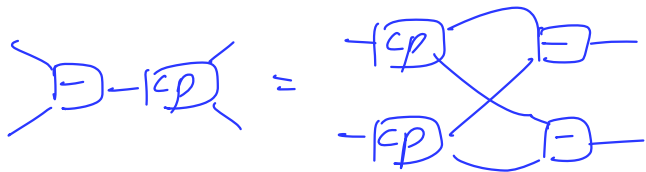
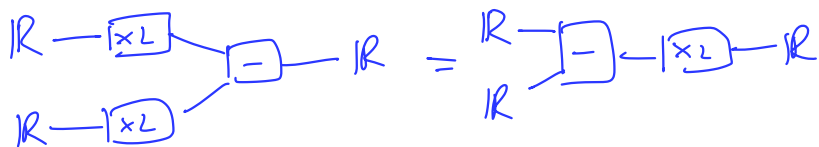
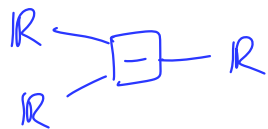
String diagrams





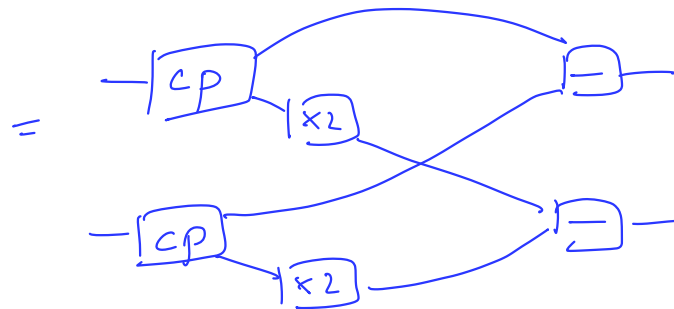
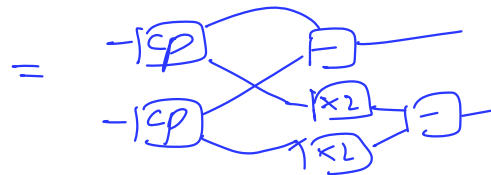
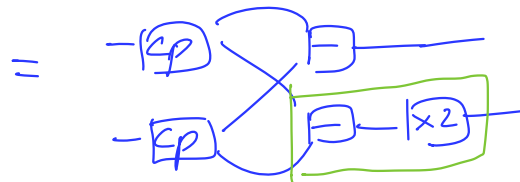
$$= ([h] \otimes id) \circ ([g] \otimes id \otimes [l]) \circ ([f] \otimes [k])$$

Diagrammatic reasoning

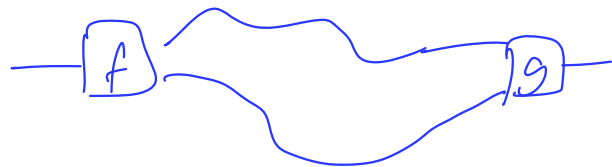
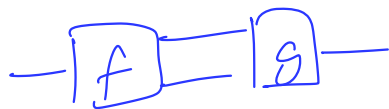


$$(x, y) \mapsto x - y \mapsto (x - y, x - y) \mapsto \begin{pmatrix} x - y \\ 2(x - y) \end{pmatrix} = \begin{pmatrix} x - y \\ 2x - 2y \end{pmatrix}$$

A square node with $-$ is connected to a square node with cp . The output of the cp node goes into a square node with $\times 2$. The entire $-$ and cp nodes are enclosed in a green box.



Isotopy

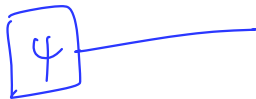


Soundness and completeness

algebra	pictures
$\otimes, \circ$	2d, no $\times$
$\otimes, \circ$, braided (topological q. comp.)	3d, 
$\otimes, \circ$, symmetry	4d, 

States, effects, scalars

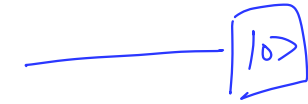
States



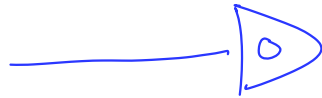
$$[\triangleleft \psi \triangleright] : (\mathbb{C}^L)^{\otimes 0} \longrightarrow (\mathbb{C}^2)^{\otimes 1}$$

$$\mathbb{C} \longrightarrow \mathbb{C}^2$$

Effects



||



postselection (Sig)

Scalars

$$[\![\alpha]\!] : (\mathbb{C}^2)^{\otimes 0} = \mathbb{C} \longrightarrow \mathbb{C} \quad \text{just a number}$$

$$[\![\alpha]\!] [\![\beta]\!]$$

$$[\![\beta]\!] [\![\alpha]\!]$$

$$\equiv$$

$$\equiv$$

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$$\langle \psi | \varphi \rangle$$

$$[\![\langle \psi | \varphi \rangle]\!] = [\![\langle \psi |]\!] \circ [\![|\varphi\rangle]\!] \\ \mathbb{C}^2 \rightarrow \mathbb{C} \quad \mathbb{C} \rightarrow \mathbb{C}^1$$

Entanglement

Entanglement

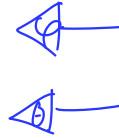
bipartite state is of the form



it is separable if



=



for some states ϕ, θ

it is entangled otherwise

Summary:

- String diagrams have (4d) graphical calculus
- Sound and complete for sequential and parallel composition
- States, effect, and scalars are derived concepts
- Entanglement makes sense at graphical level