Introduction to Quantum Programming and Semantics 2026
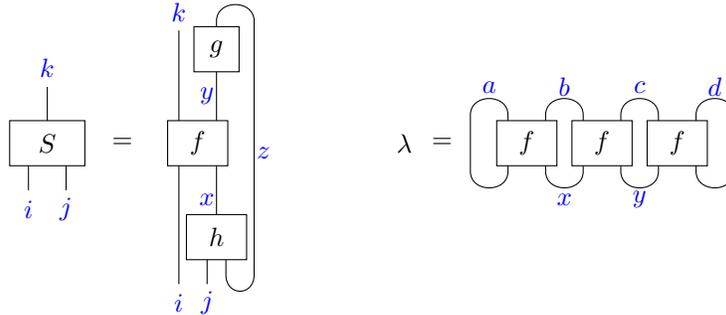## Tutorial week 4

Exercise 1

$$\Phi = \mathrm{Tr}((\mathrm{id} \otimes g) \circ (\mathrm{SWAP} \otimes \mathrm{id}) \circ (f \otimes h))$$

Exercise 2

Labelling the wires with some index names:

...we get:

$$S^k_{ij} = \sum_{xyz} f^{ky}_{ix} g^z_y h^x_{jz}$$

$$\lambda = \sum_{abcdxy} f^{ab}_{ax} f^{bc}_{xy} f^{cd}_{yd}$$

Note λ is a scalar, so all indices on the RHS are summed over.

Exercise 3

(a) The copy map $d \colon I \to I \otimes I$ is the empty picture with (once) zero wires in and (twice) zero wires out. Similarly, the identity map $I \to I$ is the empty picture with (once) zero wires in and (once) zero wires out. Hence the associativity condition holds vacuously: the left-hand side is a composition of empty pictures, as is the right-hand side. Commutativity similarly holds vacuously, because the swap map $I \otimes I \to I \otimes I$ is also the empty picture with (twice) zero wires in and (twice) zero wires out. Finally, as discarding map $e \colon I \to I$ we can also take the empty picture, making the unitality condition true in the same way.
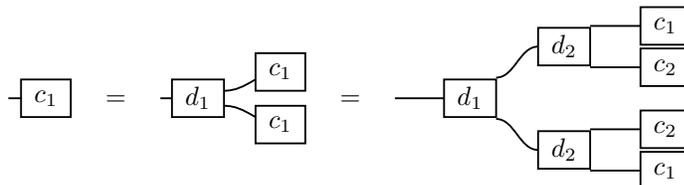
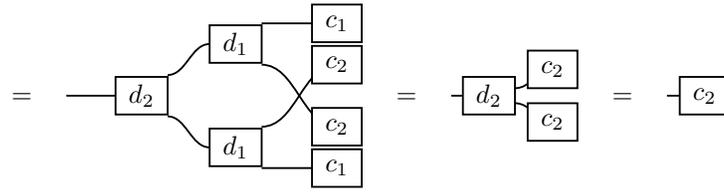(b) Specialising the definition in the question, a homomorphism $a \colon I \to A$ is a map satisfying:
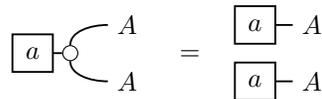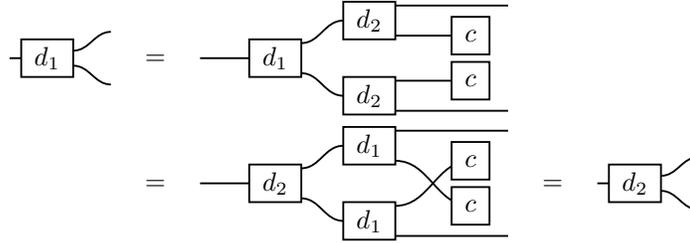

But that is exactly $f \circ a = a \otimes a$.

Exercise 4

(a)

$=$

(b) Writing c for $c_1 = c_2$:

(c) Writing d for $d_1 = d_2$: