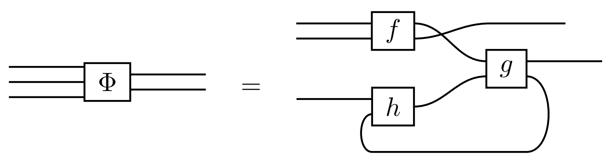
Introduction to Quantum Programming and Semantics 2026
Tutorial week 4

These exercises relate mostly to Lectures 6 and 7 on tensor networks and copying (and build on Lectures 4 and 5 on graphical calculus).

Exercise 1

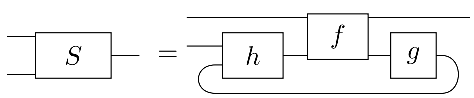
Write an algebraic expression for the following diagram using $\otimes$, $\circ$, SWAP, and $\mathrm{Tr}(-)$.

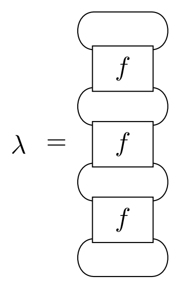


Show that it evaluates to $\Phi^{de}_{abc} = \sum_{xyz} f^{xd}_{ab} g^{ez}_{xy} h^{y}_{cz}$.

Exercise 2

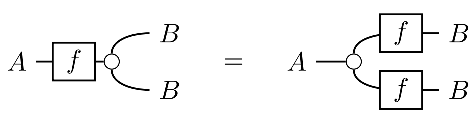
Write the following diagrams as tensor contractions, i.e. as sums over products of matrix elements f^{kl}_{ij}, etc.





Exercise 3

If systems A and B have copying maps, we say that $f\colon A \to B$ is a homomorphism when:

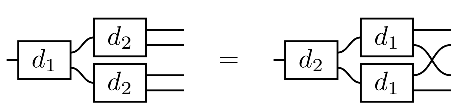


(a) Show that the empty system I (that pictorially consists of zero wires) always has a copy map that is associative, commutative, and unital.

(b) Show that a homomorphism $a\colon I \to A$ is the same thing as a state of A that is copyable in the sense that it satisfies $d \circ a = a \otimes a$, where $d\colon A \to A \otimes A$ is the copying map.

Exercise 4

Suppose you have two copy maps $d_1, d_2\colon A \to A \otimes A$ that are associative, and unital with $c_1, c_2\colon A \to I$, and that interact in the following way:



(In terms of Exercise 3: each copy map is a homomorphism for the other.)

(a) Show that $c_1 = c_2$.

(b) Show that $d_1 = d_2$.

(c) Show that d_1 is commutative.

(This is called the *Eckmann-Hilton argument*.)