Introduction to Quantum Programming and Semantics 2026
Tutorial week 5

These exercises relate to Lecture 7, 8, and 9, about copying, deleting and classical structures.

Exercise 1

Suppose that wires A and B have Frobenius structures.

(a) Show that $A \otimes B$ also carries a Frobenius structure.

(b) Show that, if A and B are classical structures, so is $A \otimes B$.

Exercise 2

This exercise is about the interdependencies of the defining properties of Frobenius structures.

(a) Show that for any maps $A \xrightarrow{d} A \otimes A$ and $A \otimes A \xrightarrow{m} A$, the Frobenius law and speciality together imply associativity for m.

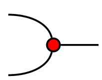
(b) Suppose that d and m satisfy the extended Frobenius law, speciality, and commutativity. Construct a map $I \xrightarrow{u} A$ satisfying unitality.

Exercise 3

Show that Z-spiders for $m, n > 0$ always correspond to matrices with rank 2. What are the other possibilities for the rank of the Z-spider if m or n is 0?

Exercise 4

The map

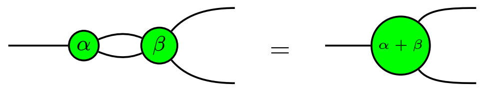


$$= \frac{1}{\sqrt{2}}\big(|0\rangle\langle 00| + |0\rangle\langle 11| + |1\rangle\langle 01| + |1\rangle\langle 10| \big)$$

is classical, in the sends that it sends Z-basis states to Z-basis states. (In fact, it is the XOR map: it sends $|00\rangle$ and $|11\rangle$ to $|0\rangle$, while $|01\rangle$ and $|10\rangle$ are mapped to $|1\rangle$.) What classical map would we get if we instead took the X-spider with 2 inputs, 1 output, and a π phase?

Exercise 6

Show that the following graphical equation is true by writing down the left-hand side as a tensor contraction and simplifying it.



Exercise 6

Prove that the following OpenQASM programs are semantically equivalent:

```
qubit a;
qubit b;
reset b;
reset a;
H a;
id a;
sy a;
result = measure a;
```

```
result = 1;
```

Here, the gate sy is interpreted as the matrix $\begin{bmatrix} \frac{1+i}{2} & \frac{-1-i}{2} \\ \frac{1+i}{2} & \frac{1+i}{2} \end{bmatrix}$.