

Introduction to Quantum Programming and Semantics 2026

Tutorial week 5

These exercises relate to Lecture 7, 8, and 9, about copying, deleting and classical structures. Recall that a spider is a family of maps that copies a fixed orthonormal basis. We call the spider copying the computational basis $\{(|0\rangle, |1\rangle)\}$ the *Z-spider*. Similarly, the spider copying the Hadamard basis $\{(|+\rangle/\sqrt{2}, |-\rangle/\sqrt{2})\}$ is called the *X-spider*.

Exercise 1

Suppose that wires A and B have Frobenius structures.

- (a) Show that $A \otimes B$ also carries a Frobenius structure.
- (b) Show that, if A and B are classical structures, so is $A \otimes B$.

Exercise 2

This exercise is about the interdependencies of the defining properties of Frobenius structures.

- (a) Show that for any maps $A \xrightarrow{d} A \otimes A$ and $A \otimes A \xrightarrow{m} A$, the extended Frobenius law and speciality together imply associativity for m .
- (b) Suppose that d and m satisfy the extended Frobenius law, speciality, and commutativity. Assume that cups and caps are available. Construct a map $I \xrightarrow{u} A$ satisfying unitality.

Exercise 3

Show that Z -spiders for $m, n > 0$ always correspond to matrices with rank 2. What are the other possibilities for the rank of the Z -spider if m or n is 0?

Exercise 4

The map

$$\text{Circuit diagram: a wire enters from the left, splits into two wires that cross, and then merge back into one wire.} = \frac{1}{\sqrt{2}} (|0\rangle\langle 00| + |0\rangle\langle 11| + |1\rangle\langle 01| + |1\rangle\langle 10|)$$

is classical, in the sense that it sends Z -basis states to Z -basis states. (In fact, it is the XOR map: it sends $|00\rangle$ and $|11\rangle$ to $|0\rangle$, while $|01\rangle$ and $|10\rangle$ are mapped to $|1\rangle$.) What classical map would we get if we instead took the X -spider with 2 inputs, 1 output, and a π phase?

Exercise 6

Show that the following graphical equation is true by writing down the left-hand side as a tensor contraction and simplifying it.

$$\text{Circuit diagram: a wire enters from the left, passes through a green circle labeled } \alpha, \text{ then a green circle labeled } \beta, \text{ and then splits into two wires.} = \text{Circuit diagram: a wire enters from the left, passes through a single green circle labeled } \alpha + \beta, \text{ and then splits into two wires.}$$

Exercise 6

Prove that the following OpenQASM programs are semantically equivalent:

```
qubit a;                                result = 1;
qubit b;
reset b;
reset a;
H a;
id a;
sy a;
result = measure a;
```

Here, the gate `sy` is interpreted as the matrix $\begin{bmatrix} \frac{1+i}{2} & \frac{-1-i}{2} \\ \frac{1+i}{2} & \frac{1+i}{2} \end{bmatrix}$.