Introduction to Quantum Programming and Semantics 2026
Tutorial week 6

Exercise 1

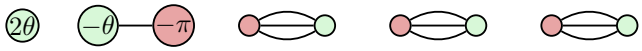
$$\tfrac{1}{\sqrt{2}}\begin{pmatrix}1&0&0&1&0&0&0&0\\0&1&1&0&0&0&0&0\\0&0&0&0&1&0&0&1\\0&0&0&0&0&1&1&0\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} = \tfrac{1}{\sqrt{2}}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$$

Exercise 2

(a) 

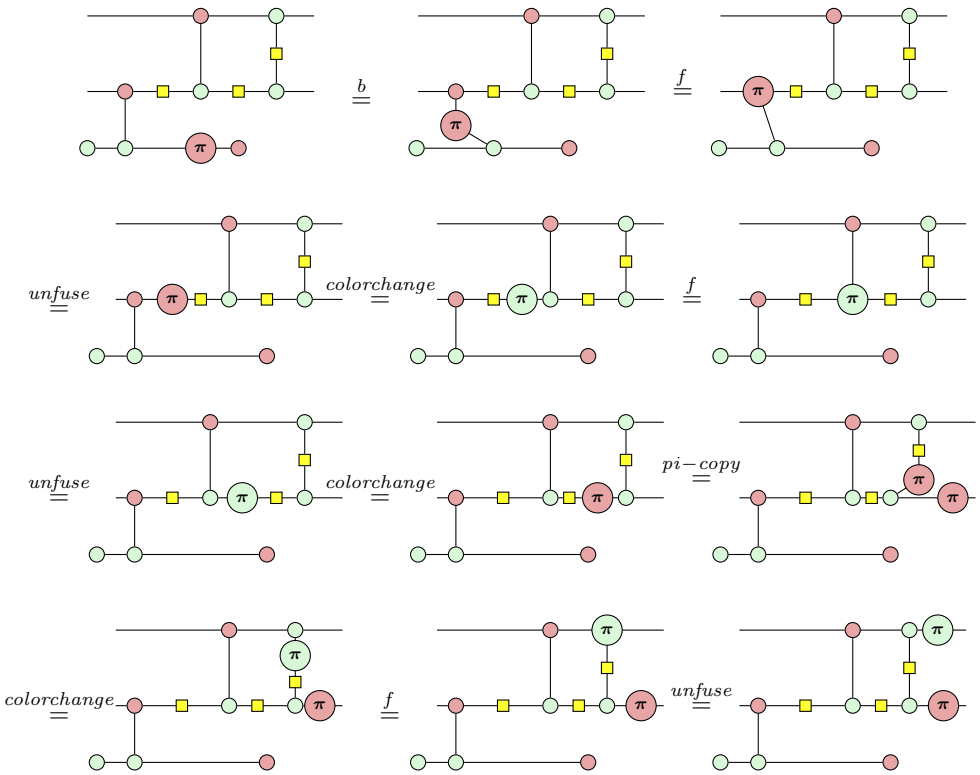
(b) 

(c) 

(d) 

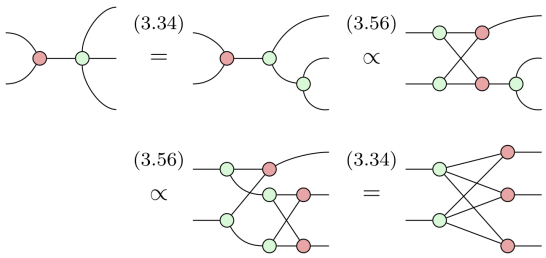
(e) First fix a k such that for $z' = 1/\sqrt{2}^k z$ we have $|z'| \leq 1$. Then we can find phases α, β such that $z' = e^{i\alpha}\cos\beta$. Since we know how to write these three components as diagrams, we are then done.

Exercise 3



Exercise 4

For $m = n = 2$ this is exactly the strong complementarity rule. For $m = 1$ or $n = 1$ this follows trivially by adding and removing identities. For $m = 0$ or $n = 0$ this is exactly the state-copy rule. Here is the induction step with $n = 2$ and $m = 3$:



Exercise 5