

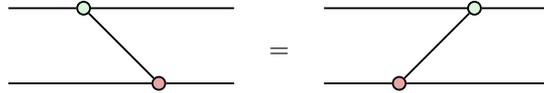
# Introduction to Quantum Programming and Semantics 2026

## Tutorial week 6

These exercises relate to Lecture 10 and 11 on complementarity and ZX calculus rewriting.

### Exercise 1

Prove the following graphical equation by computing the matrix interpretations of both sides.



### Exercise 2

In the notes and lectures we ignore scalar factors in ZX-diagrams, but we can represent any scalar we want with a ZX-diagram. For instance, we have:

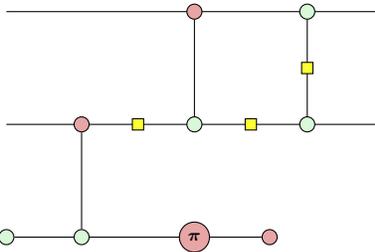
$$\begin{array}{ll}
 \llbracket \bigcirc \rrbracket = 2 & \llbracket \text{red circle} \text{---} \text{white circle} \rrbracket = \sqrt{2} \\
 \llbracket \text{white circle} \rrbracket = 0 & \llbracket \text{red circle} \text{---} \text{white circle} \text{---} \text{red circle} \rrbracket = \sqrt{2}e^{i\alpha} \\
 \llbracket \text{white circle} \rrbracket = 1 + e^{i\alpha} & \llbracket \text{red circle} \text{---} \text{white circle} \text{---} \text{red circle} \text{---} \text{white circle} \rrbracket = \frac{1}{\sqrt{2}}
 \end{array}
 \quad (*)$$

By combining the diagrams from (\*), find a ZX-diagram to represent the following scalars  $z$ :

- (a)  $z = -1$ .
- (b)  $z = e^{i\theta}$  for any  $\theta$ .
- (c)  $z = \frac{1}{2}$ .
- (d)  $z = \cos \theta$  for any value  $\theta$ .
- (e) Describe a systematic way to construct the ZX-diagram for any complex number  $z$ .

### Exercise 3

Using ZX-calculus rewrites, help the poor trapped  $\pi$  phase find it's way to an exit (i.e. an output).



Note that it might be leaving with friends.

### Exercise 4

Prove the following rule by induction on the number of input and output wires:



Here, the ZX diagram on the right is the full connected bipartite graph.

### Exercise 5

Prove the following quantum circuit identity by translating to ZX diagrams and rewriting:

