

Introduction to Quantum Programming and Semantics 2026

Tutorial week 8

Exercise 1

Compute

$$|H'\rangle = (e^{i\frac{\pi}{8}} + e^{-i\frac{\pi}{8}})|0\rangle - i(e^{i\frac{\pi}{8}} - e^{-i\frac{\pi}{8}})|1\rangle.$$

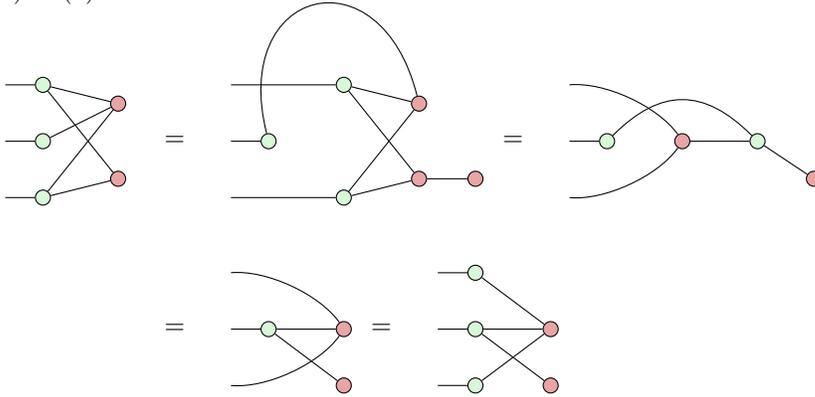
It then follows that

$$e^{-i\frac{\pi}{8}}HS|H'\rangle = \sqrt{2}|0\rangle + \sqrt{2}e^{-i\frac{\pi}{4}}|1\rangle.$$

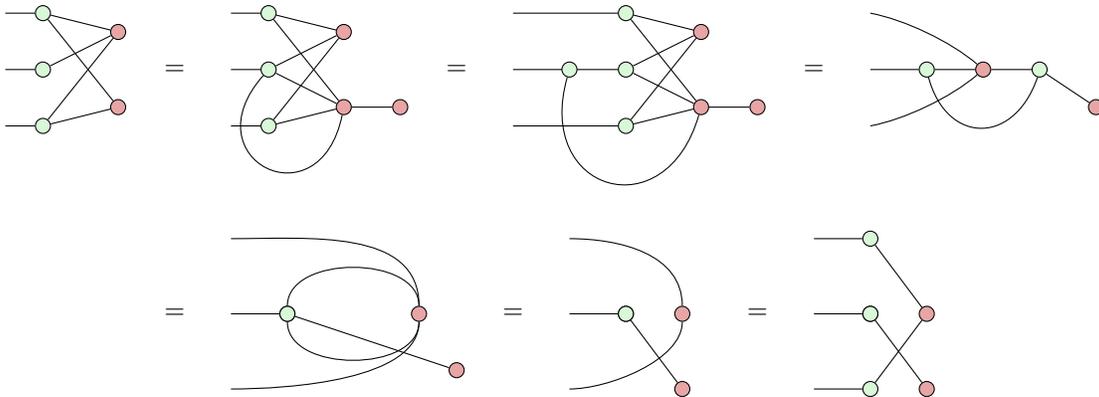
Hence $S^\dagger HT^\dagger|+\rangle = |H\rangle$ up to global phase.

Exercise 2

(1) = (2)



(1) = (3)



Exercise 3

Composing the normal form with S again gives a normal form by spider fusion. When we compose a H we make a case distinction on b . If $b = 0$, then a and c fuse, and we can just decompose the H into its ZXZ Euler decomposition to get a normal form again after spider fusion. We can do something similar when $b = 2$, by moving the X gate out of the way first. If $b = 1$, we realise the

normal form can also be written as $Z((c-1)\frac{\pi}{2})HZ((a-1)\frac{\pi}{2})$ by using the Euler decomposition in reverse. The result is then easily proven. The same when $b = 3$.

Exercise 4

The circuits are:

- (a) $X = HZH = HSSH$
- (b) HX
- (c) $CX(H \otimes 1)$
- (d) $(1 \otimes X)CX(H \otimes 1)$
- (e) $CX_{1,3}CX_{1,2}(H \otimes 1 \otimes 1)$