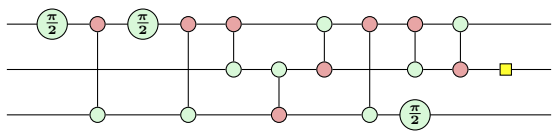
Introduction to Quantum Programming and Semantics 2026
Tutorial week 9

These exercises relate to Lecture 15 on circuit optimisation and Lecture 16 on Pauli gadgets.

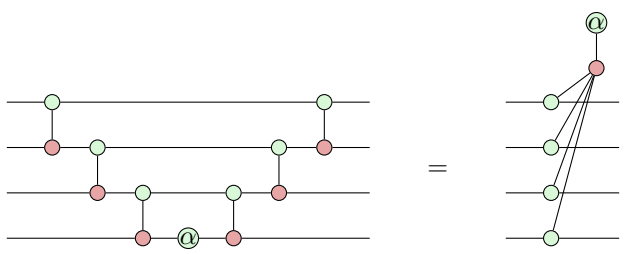
Exercise 1

Simplify the following circuit to a diagram that has no internal spiders with a $\pm\frac{\pi}{2}$ phase or pairs of internal spiders with a 0 or a π phase.



Exercise 2

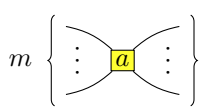
Show that phase gadgets have the following decomposition:



(Hint: Use equations (7.13), (7.14), and (7.15) from [KW].)

Exercise 3

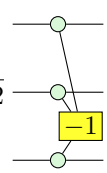
Define an *H-box* as follows:



$$= \sum a^{i_1 \ldots i_m j_1 \ldots j_n} \left| j_1 \ldots j_n \right\rangle \left\langle i_1 \ldots i_m \right|$$

Verify the following equation by plugging in computational basis states and checking that it gives the correct phase:

$$CCZ = \sqrt{2}$$



$(*)$

Exercise 4

Show by ZX diagram rewriting that the CCZ gate as defined in $(*)$ is its own inverse (up to scalar normalisation).