Abstract Syntax	DL Syntax	Semantics
Class(A)	A	$\mathsf{A}^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
Class(owl:Thing)	$\top$	$\top^{\mathcal{I}} = \Delta^{\mathcal{I}}$
Class(owl:Nothing)	$\bot$	$\bot^{\mathcal{I}} = \emptyset$
intersectionOf($C_1, C_2, \ldots$)	$C_1 \sqcap C_2$	$(C_1 \sqcap C_2)^{\mathcal{I}} = C_1^{\mathcal{I}} \cap C_2^{\mathcal{I}}$
unionOf($C_1, C_2, \ldots$)	$C_1 \sqcup C_2$	$(C_1 \sqcup C_2)^{\mathcal{I}} = C_1^{\mathcal{I}} \cup C_2^{\mathcal{I}}$
complementOf(C)	$\neg C$	$(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
oneOf($o_1, o_2, \ldots$)	$\{o_1\} \sqcup \{o_2\}$	$(\{o_1\} \sqcup \{o_2\})^{\mathcal{I}} = \{o_1{}^{\mathcal{I}}, o_2{}^{\mathcal{I}}\}$
restriction(R someValuesFrom(C))	$\exists R.C$	$(\exists R.C)^{\mathcal{I}} = \{x \mid \exists y.\langle x, y\rangle \in R^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\}$
restriction(R allValuesFrom(C))	$\forall R.C$	$(\forall R.C)^{\mathcal{I}} = \{x \mid \forall y.\langle x, y\rangle \in R^{\mathcal{I}} \rightarrow y \in C^{\mathcal{I}}\}$
restriction(R hasValue(o))	$\exists R.\{o\}$	$(\exists R.\{o\})^{\mathcal{I}} = \{x \mid \langle x, \mathsf{o}^{\mathcal{I}}\rangle \in R^{\mathcal{I}}\}$
restriction(R minCardinality(m))	$\geqslant mR$	$(\geqslant mR)^{\mathcal{I}} = \{x \mid \sharp\{y.\langle x, y\rangle \in R^{\mathcal{I}}\} \geq m\}$
restriction(R maxCardinality(m))	$\leqslant mR$	$(\leqslant mR)^{\mathcal{I}} = \{x \mid \sharp\{y.\langle x, y\rangle \in R^{\mathcal{I}}\} \leq m\}$
restriction(T someValuesFrom(u))	$\exists T.u$	$(\exists T.u)^{\mathcal{I}} = \{x \mid \exists t.\langle x, t\rangle \in T^{\mathcal{I}} \wedge t \in u^{\mathbf{D}}\}$
restriction(T allValuesFrom(u))	$\forall T.u$	$(\forall T.u)^{\mathcal{I}} = \{x \mid \exists t.\langle x, t\rangle \in T^{\mathcal{I}} \rightarrow t \in u^{\mathbf{D}}\}$
restriction(T hasValue(w))	$\exists T.\{w\}$	$(\exists T.\{w\})^{\mathcal{I}} = \{x \mid \langle x, w^{\mathbf{D}}\rangle \in T^{\mathcal{I}}\}$
restriction(T minCardinality(m))	$\geqslant mT$	$(\geqslant mT)^{\mathcal{I}} = \{x \mid \sharp\{t \mid \langle x, t\rangle \in T^{\mathcal{I}}\} \geq m\}$
restriction(T maxCardinality(m))	$\leqslant mT$	$(\leqslant mT)^{\mathcal{I}} = \{x \mid \sharp\{t \mid \langle x, t\rangle \in T^{\mathcal{I}}\} \leq m\}$
ObjectProperty(S)	S	$S^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
ObjectProperty(S' inverseOf(S))	S^-	$(S^-)^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
DatatypeProperty(T)	T	$T^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta_{\mathbf{D}}$

Abstract Syntax	DL Syntax	Semantics
Class(A partial $C_1 \ \ldots\ C_n$)	$A \sqsubseteq C_1 \sqcap \ldots \sqcap C_n$	$A^{\mathcal{I}} \subseteq C_1^{\mathcal{I}} \cap \ldots \cap C_n^{\mathcal{I}}$
Class(A complete $C_1 \ \ldots\ C_n$)	$A \equiv C_1 \sqcap \ldots \sqcap C_n$	$A^{\mathcal{I}} = C_1^{\mathcal{I}} \cap \ldots \cap C_n^{\mathcal{I}}$
EnumeratedClass(A $\mathsf{o}_1 \ \ldots\ \mathsf{o}_n$)	$A \equiv \{\mathsf{o}_1\} \sqcup \ldots \sqcup \{\mathsf{o}_n\}$	$A^{\mathcal{I}} = \{\mathsf{o}_1{}^{\mathcal{I}}, \ldots, \mathsf{o}_n^{\mathcal{I}}\}$
SubClassOf(C_1, C_2)	$C_1 \sqsubseteq C_2$	$C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}}$
EquivalentClasses($C_1 \ \ldots\ C_n$)	$C_1 \equiv \ldots \equiv C_n$	$C_1^{\mathcal{I}} = \ldots = C_n^{\mathcal{I}}$
DisjointClasses($C_1 \ \ldots\ C_n$)	$C_i \sqsubseteq \neg C_j$, $(1 \leq i < j \leq n)$	$C_1^{\mathcal{I}} \cap C_n^{\mathcal{I}} = \emptyset$, $(1 \leq i < j \leq n)$
SubPropertyOf(R_1, R_2)	$R_1 \sqsubseteq R_2$	$R_1^{\mathcal{I}} \subseteq R_2^{\mathcal{I}}$
EquivalentProperties($R_1 \ \ldots\ R_n$)	$R_1 \equiv \ldots \equiv R_n$	$R_1^{\mathcal{I}} = \ldots = R_n^{\mathcal{I}}$
ObjectProperty(R super(R_1) ... super(R_n)	$R \sqsubseteq R_i$	$R^{\mathcal{I}} \subseteq R_i^{\mathcal{I}}$
domain(C_1) ... domain(C_k)	$\geqslant 1R \sqsubseteq C_i$	$R^{\mathcal{I}} \subseteq C_i^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
range(C_1) ... range(C_h)	$\top \sqsubseteq \forall R.C_i$	$R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times C_i^{\mathcal{I}}$
[Symmetric]	$R \equiv R^-$	$R^{\mathcal{I}} = (R^-)^{\mathcal{I}}$
[Functional]	$\mathsf{Func}(R)$	$\{\langle x,y\rangle \mid \sharp\{y.\langle x,y\rangle \in R^{\mathcal{I}}\} \leq 1\}$
[InverseFunctional]	$\mathsf{Func}(R^-)$	$\{\langle x,y\rangle \mid \sharp\{y.\langle x,y\rangle \in (R^-)^{\mathcal{I}}\} \leq 1\}$
[Transitive])	$\mathsf{Trans}(R)$	$R^{\mathcal{I}} = (R^{\mathcal{I}})^+$
AnnotationProperty(R)		
Individual(o type(C_1) ... type(C_n)	$\mathsf{o} : C_i, 1 \leq i \leq n$	$\mathsf{o}^{\mathcal{I}} \in C_i^{\mathcal{I}}, 1 \leq i \leq n$
value(R_1, o_1) ... value(R_n, o_n)	$\langle \mathsf{o}, \mathsf{o}_i \rangle : R_i, 1 \leq i \leq n$	$\langle \mathsf{o}^{\mathcal{I}}, \mathsf{o}_i^{\mathcal{I}} \rangle \in R_i^{\mathcal{I}}, 1 \leq i \leq n$
SameIndividual($\mathsf{o}_1 \ \ldots\ \mathsf{o}_n$)	$\mathsf{o}_1 = \ldots = \mathsf{o}_n$	$\mathsf{o}_1^{\mathcal{I}} = \ldots = \mathsf{o}_n^{\mathcal{I}}$
DifferentIndividuals($\mathsf{o}_1 \ \ldots\ \mathsf{o}_n$)	$\mathsf{o}_i \neq \mathsf{o}_j, 1 \leq i < j \leq n$	$\mathsf{o}_i^{\mathcal{I}} \neq \mathsf{o}_j^{\mathcal{I}}, 1 \leq i < j \leq n$

The $\sqcap$-rule

Condition: $\mathcal{A}$ contains $a:(C \sqcap D)$, but not both $a:C$ and $a:D$

Action: $\mathcal{A} \longrightarrow \mathcal{A} \cup \{a:C, a:D\}$

The $\sqcup$-rule

Condition: $\mathcal{A}$ contains $a:(C \sqcup D)$, but neither $a:C$ nor $a:D$

Action: $\mathcal{A} \longrightarrow \mathcal{A} \cup \{a:X\}$ for some $X \in \{C, D\}$

The $\exists$-rule

Condition: $\mathcal{A}$ contains $a:(\exists r.C)$, but there is no b with $\{(a,b):r, b:C\} \subseteq \mathcal{A}$

Action: $\mathcal{A} \longrightarrow \mathcal{A} \cup \{(a,d):r, d:C\}$ where d is new in $\mathcal{A}$

The $\forall$-rule

Condition: $\mathcal{A}$ contains $a:(\forall r.C)$ and $(a,b):r$, but not $b:C$

Action: $\mathcal{A} \longrightarrow \mathcal{A} \cup \{b:C\}$

NF0	$\widehat{D} \sqsubseteq \widehat{E}$	$\longrightarrow$	$\widehat{D} \sqsubseteq A,\ \ A \sqsubseteq \widehat{E}$
NF1$_r$	$C \sqcap \widehat{D} \sqsubseteq B$	$\longrightarrow$	$\widehat{D} \sqsubseteq A,\ \ C \sqcap A \sqsubseteq B$
NF1$_\ell$	$\widehat{D} \sqcap C \sqsubseteq B$	$\longrightarrow$	$\widehat{D} \sqsubseteq A,\ \ A \sqcap C \sqsubseteq B$
NF2	$\exists r.\widehat{D} \sqsubseteq B$	$\longrightarrow$	$\widehat{D} \sqsubseteq A,\ \ \exists r.A \sqsubseteq B$
NF3	$B \sqsubseteq \exists r.\widehat{D}$	$\longrightarrow$	$A \sqsubseteq \widehat{D},\ \ B \sqsubseteq \exists r.A$
NF4	$B \sqsubseteq D \sqcap E$	$\longrightarrow$	$B \sqsubseteq D,\ \ B \sqsubseteq E$

where C, D, E denote arbitrary $\mathcal{EL}$ concepts,
$\widehat{D}, \widehat{E}$ denote $\mathcal{EL}$ concepts that are neither concept names nor $\top$,
B is a concept name, and
A is a new concept name.

$$\text{CR1}\quad \frac{}{A \sqsubseteq A} \qquad \text{CR2}\quad \frac{}{A \sqsubseteq \top}$$

$$\text{CR3}\quad \frac{A_1 \sqsubseteq A_2 \quad A_2 \sqsubseteq A_3}{A_1 \sqsubseteq A_3} \qquad \text{CR4}\quad \frac{A \sqsubseteq A_1 \quad A \sqsubseteq A_2 \quad A_1 \sqcap A_2 \sqsubseteq B}{A \sqsubseteq B}$$

$$\text{CR5}\quad \frac{A \sqsubseteq \exists r.A_1 \quad A_1 \sqsubseteq B_1 \quad \exists r.B_1 \sqsubseteq B}{A \sqsubseteq B}$$