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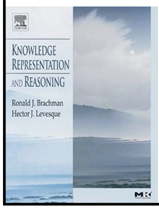
INFR11215 Knowledge Graphs

Expressing Knowledge

Jeff Z. Pan

http://knowledge-representation.org/j.z.pan/



[Reading: Brachman and Levesque, chapter 3]

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Lecture Outline

- Motivation
- Basic Facts and Terminological Facts
- Complex Facts and Entailment
- Practical

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Motivations

- Description Logics is the chosen family of logics for KR
- How to represent
  - There are known knowns
  - There are also unknown knowns
  - We also know there are known unknowns
  - But there are also unknown unknowns

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RDF: Modern Standard for Semantic Networks

- RDF: Resource Description Framework
- Basic building block: Subject-property-value triple
  - It is called a statement
  - E.g. Tom teaches CS3025 is a statement
- RDF (Resource Description Framework) has been given a syntax in XML
  - This syntax inherits the benefits of XML
  - Other syntactic representations of RDF possible
    - such as Notation 3 (N3) syntax: [Subject property value .]

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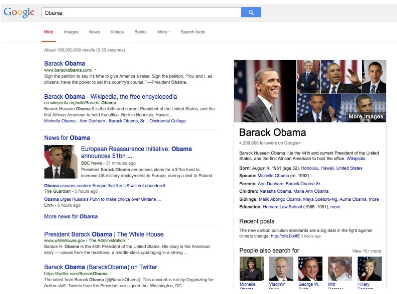
RDF Statements

- Statements assert the properties of resources
- A statement is an resource-property-value triple
  - It consists of a resource, a property, and a value
- Values can be resources or **literals**
  - Type literals: “15”^^xsd:integer
  - Plain literals (strings): “Tom”



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RDF Schema

- RDF Schema (RDFS)
  - Allow users to introduce vocabulary
- Key meta classes for declarations:
  - rdfs:Class, rdfs:Resource, rdfs:Property
- Key meta properties for basic axioms:
  - rdf:type, rdfs:subClassOf, rdfs:subPropertyOf, rdfs:domain, rdfs:range

- Examples:

[csd:Tom rdf:type csd:Student .]
[csd:Tom csd:takeCourse csd:3025 .]
[csd:Lecturer rdfs:subClassOf csd:AssistantProf .]
[csd:takeCourse rdfs:subPropertyOf csd:receiveTraining .]
[csd:takeCourse rdfs:domain csd:Student .]
[csd:takeCourse rdfs:range csd:Course .]

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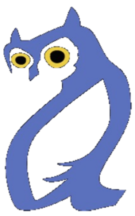
Symbols in Description Logics



- Non-logical symbols
  - Predicates
    - Classes / Concpets (unary predicates)
    - Properties / Roles / Relation (binary predicates)
  - Constants (0-ary function symbols)
    - Objects / Individuals / Entity
- Logical symbols
  - Punctuation: (,), .
  - Connectives: $\neg, \sqcap, \sqcup, \forall, \exists, \leq, \geq$
  - No variable symbols

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Basic Facts

- Relating individuals with named classes and named properties
- Class Assertions
  - DL syntax: e:A, or A(e)
  - RDF Notation 3 (N3) syntax: [e rdf:type A .]
  - FOL syntax: A(e)
- Property Assertions
  - DL syntax: (e1,e2):r, or r(e1, e2)
  - RDF N3 syntax: [e1 r e2 .]
  - FOL syntax: r(e1,e2)
- Equality / Inequality assertions
  - e1 = e2, e1 ≠ e2

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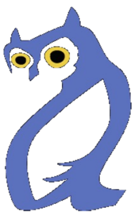
Terminological Facts/Class Axioms



- They are also called axioms, or schema axioms, to define the class and properties
- SubClassOf axioms
  - DL syntax: $C1 \sqsubseteq C2$
  - FOL syntax: $\forall x$ [C1(x) -> C2(x)]
- Equivalent Class axioms
  - DL syntax: $C1 \equiv C2$
  - FOL syntax: $\forall x$ [C1(x) <-> C2(x)]
- Enumerated Class axioms
  - DL syntax: $A \equiv$ {e1, ...,en}
  - FOL syntax: $\forall x$ [A(x) -> (x=e1 $\vee$ . . . $\vee$ x=en)]

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Terminological Facts/Class Axioms (2)

- They are also called axioms, or schema axioms
- Disjoint Class axioms
  - DL syntax: $C1 \sqsubseteq \neg C2$
  - FOL syntax: $\forall x[C1(x) \rightarrow \neg C2(x)]$
- Exhaustive Class axioms
  - DL syntax: $C \sqsubseteq C1 \sqcup \ldots \sqcup Cn$
  - FOL syntax: $\forall x[\, C(x) \rightarrow (C1(x) \vee \ldots \vee Cn(x)\,]$

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Terminological Facts/Property Axioms

- SubPropertyOf axioms
  - DL syntax: $r1 \sqsubseteq r2$
  - FOL syntax: $\forall x,y[r1(x,y) \rightarrow r2(x,y)]$
- Property Domain axioms
  - DL syntax: $\exists r \sqsubseteq C$
  - FOL syntax: $\forall x[\exists y.r(x,y) \rightarrow C(x)]$
- Property Range axioms
  - DL syntax: $\exists r^{-} \sqsubseteq C$
  - FOL syntax: $\forall x[\exists y.r(y,x) \rightarrow C(x)]$
- Inverse Property axioms
  - DL syntax: $r1 \equiv r2^{-}$
  - FOL syntax: $\forall x,y[r1(y,x) \leftrightarrow r2(x,y)]$

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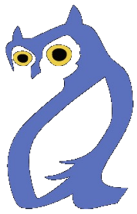
Terminological Facts/Property Axioms (2)



- Symmetric axioms
  - DL syntax: r ≅ r⁻
  - FOL syntax: ∀x,y[r(x,y) -> x=y]
- Functional Property axioms
  - DL syntax: Func(r)
  - FOL syntax: ∀x,y,z[r(x,y)∧r(x,z) -> y=z]
- Inverse Functional Property axioms
  - DL syntax: Func(r⁻)
  - FOL syntax: ∀x,y,z[r(y,x)∧r(z,x) -> y=z]
- Transitive Property axioms
  - DL syntax: Trans(r)
  - FOL syntax: ∀x,y,z[r(x,y)∧r(y,z) -> r(x,z)]

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OWL Axioms

Abstract Syntax	DL Syntax	Semantics
Class(A partial $C_1 \ldots C_n$)	$A \sqsubseteq C_1 \sqcap \ldots \sqcap C_n$	$A^{\mathcal{I}} \subseteq C_1^{\mathcal{I}} \cap \ldots \cap C_n^{\mathcal{I}}$
Class(A complete $C_1 \ldots C_n$)	$A \equiv C_1 \sqcap \ldots \sqcap C_n$	$A^{\mathcal{I}} = C_1^{\mathcal{I}} \cap \ldots \cap C_n^{\mathcal{I}}$
EnumeratedClass(A $o_1 \ldots o_n$)	$A \equiv \{o_1\} \sqcup \ldots \sqcup \{o_n\}$	$A^{\mathcal{I}} = \{o_1^{\mathcal{I}}, \ldots, o_n^{\mathcal{I}}\}$
SubClassOf(C_1, C_2)	$C_1 \sqsubseteq C_2$	$C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}}$
EquivalentClasses($C_1 \ldots C_n$)	$C_1 \equiv \ldots \equiv C_n$	$C_1^{\mathcal{I}} = \ldots = C_n^{\mathcal{I}}$
DisjointClasses($C_1 \ldots C_n$)	$C_i \sqsubseteq \neg C_j$, $(1 \le i < j \le n)$	$C_1^{\mathcal{I}} \cap C_n^{\mathcal{I}} = \emptyset$, $(1 \le i < j \le n)$
SubPropertyOf(R_1, R_2)	$R_1 \sqsubseteq R_2$	$R_1^{\mathcal{I}} \subseteq R_2^{\mathcal{I}}$
EquivalentProperties($R_1 \ldots R_n$)	$R_1 \equiv \ldots \equiv R_n$	$R_1^{\mathcal{I}} = \ldots = R_n^{\mathcal{I}}$
ObjectProperty(R super(R_1) ... super(R_n)	$R \sqsubseteq R_i$	$R^{\mathcal{I}} \subseteq R_i^{\mathcal{I}}$
domain(C_1) ... domain(C_k)	$\geqslant 1R \sqsubseteq C_i$	$R^{\mathcal{I}} \subseteq C_i^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
range(C_1) ... range(C_h)	$\top \sqsubseteq \forall R.C_i$	$R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times C_i^{\mathcal{I}}$
[Symmetric]	$R \equiv R^-$	$R^{\mathcal{I}} = (R^-)^{\mathcal{I}}$
[Functional]	Func(R)	$\{\langle x,y\rangle \mid \sharp\{y.\langle x,y\rangle \in R^{\mathcal{I}}\} \le 1\}$
[InverseFunctional]	Func(R^-)	$\{\langle x,y\rangle \mid \sharp\{y.\langle x,y\rangle \in (R^-)^{\mathcal{I}}\} \le 1\}$
[Transitive])	Trans(R)	$R^{\mathcal{I}} = (R^{\mathcal{I}})^+$
AnnotationProperty(R)		
Individual(o type(C_1) ... type(C_n)	$o : C_i, 1 \le i \le n$	$o^{\mathcal{I}} \in C_i^{\mathcal{I}}, 1 \le i \le n$
value(R_1, o_1) ... value(R_n, o_n)	$\langle o, o_i\rangle : R_i, 1 \le i \le n$	$\langle o^{\mathcal{I}}, o_i^{\mathcal{I}}\rangle \in R_i^{\mathcal{I}}, 1 \le i \le n$
SameIndividual($o_1 \ldots o_n$)	$o_1 = \ldots = o_n$	$o_1^{\mathcal{I}} = \ldots = o_n^{\mathcal{I}}$
DifferentIndividuals($o_1 \ldots o_n$)	$o_i \ne o_j, 1 \le i < j \le n$	$o_i^{\mathcal{I}} \ne o_j^{\mathcal{I}}, 1 \le i < j \le n$

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OWL Descriptions

- DLs supports complex facts, mainly by the use of complex class descriptions, including oneOf, i.e,, {e1,...,en}

Abstract Syntax	DL Syntax	Semantics
Class(A)	A	$\mathsf{A}^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
Class(owl:Thing)	$\top$	$\top^{\mathcal{I}} = \Delta^{\mathcal{I}}$
Class(owl:Nothing)	$\bot$	$\bot^{\mathcal{I}} = \emptyset$
intersectionOf($C_1, C_2, \ldots$)	$C_1 \sqcap C_2$	$(C_1 \sqcap C_2)^{\mathcal{I}} = C_1^{\mathcal{I}} \cap C_2^{\mathcal{I}}$
unionOf($C_1, C_2, \ldots$)	$C_1 \sqcup C_2$	$(C_1 \sqcup C_2)^{\mathcal{I}} = C_1^{\mathcal{I}} \cup C_2^{\mathcal{I}}$
complementOf(C)	$\neg C$	$(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
oneOf($o_1, o_2, \ldots$)	$\{o_1\} \sqcup \{o_2\}$	$(\{o_1\} \sqcup \{o_2\})^{\mathcal{I}} = \{o_1^{\mathcal{I}}, o_2^{\mathcal{I}}\}$
restriction(R someValuesFrom(C))	$\exists R.C$	$(\exists R.C)^{\mathcal{I}} = \{x \mid \exists y.\langle x,y\rangle \in R^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\}$
restriction(R allValuesFrom(C))	$\forall R.C$	$(\forall R.C)^{\mathcal{I}} = \{x \mid \forall y.\langle x,y\rangle \in R^{\mathcal{I}} \rightarrow y \in C^{\mathcal{I}}\}$
restriction(R hasValue(o))	$\exists R.\{o\}$	$(\exists R.\{o\})^{\mathcal{I}} = \{x \mid \langle x, \mathsf{o}^{\mathcal{I}}\rangle \in R^{\mathcal{I}}\}$
restriction(R minCardinality(m))	$\geqslant mR$	$(\geqslant mR)^{\mathcal{I}} = \{x \mid \sharp\{y.\langle x,y\rangle \in R^{\mathcal{I}}\} \geq m\}$
restriction(R maxCardinality(m))	$\leqslant mR$	$(\leqslant mR)^{\mathcal{I}} = \{x \mid \sharp\{y.\langle x,y\rangle \in R^{\mathcal{I}}\} \leq m\}$
restriction(T someValuesFrom(u))	$\exists T.u$	$(\exists T.u)^{\mathcal{I}} = \{x \mid \exists t.\langle x,t\rangle \in T^{\mathcal{I}} \wedge t \in u^{\mathbf{D}}\}$
restriction(T allValuesFrom(u))	$\forall T.u$	$(\forall T.u)^{\mathcal{I}} = \{x \mid \exists t.\langle x,t\rangle \in T^{\mathcal{I}} \rightarrow t \in u^{\mathbf{D}}\}$
restriction(T hasValue(w))	$\exists T.\{w\}$	$(\exists T.\{w\})^{\mathcal{I}} = \{x \mid \langle x, w^{\mathbf{D}}\rangle \in T^{\mathcal{I}}\}$
restriction(T minCardinality(m))	$\geqslant mT$	$(\geqslant mT)^{\mathcal{I}} = \{x \mid \sharp\{t \mid \langle x,t\rangle \in T^{\mathcal{I}}\} \geq m\}$
restriction(T maxCardinality(m))	$\leqslant mT$	$(\leqslant mT)^{\mathcal{I}} = \{x \mid \sharp\{t \mid \langle x,t\rangle \in T^{\mathcal{I}}\} \leq m\}$
ObjectProperty(S)	S	$S^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
ObjectProperty(S' inverseOf(S))	S^{-}	$(S^{-})^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
DatatypeProperty(T)	T	$T^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta_{\mathbf{D}}$

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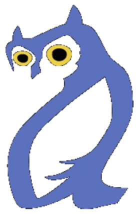
Class Descriptions



- Class descriptions
  - $C \sqcap D$: $C(x) \wedge D(x)$
  - $C \sqcup D$: $C(x) \vee D(x)$
  - $\neg C$: $\neg C(x)$
  - $\{o1,\ldots,on\}$: $o1 \vee \ldots, \vee o2$
  - Restriction (next slide)
- Example class descriptions
  - Burgundy $\sqcap$ Whitewine
  - Male $\sqcup$ Female
  - $\neg$Frenchwine
  - {White Rose Red}

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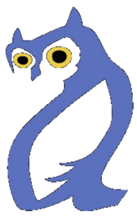
Class Descriptions (2)



- Restrictions
  - $\exists R.C$: $\exists y.R(x,y) \wedge C(y)$
  - $\forall R.C$: $\forall y.R(x,y) \to C(y)$
  - $\geq nR$: $\exists y1,\ldots,yn.R(x,y1) \wedge \ldots \wedge R(x,yn)$
  - $\leq nR$: $\forall y_1,\ldots,y_{n+1}.R(x,y_1) \wedge \ldots \wedge R(x,y_{n+1}) \to \bigvee_{1 \leq i < j \leq n+1} y_i = y_j$
  - $=nR$
- Example restrictions
  - $\exists$hasBase.PizzaBase
  - $\forall$eat.Plant:
  - $\geq$5hasFinger:
  - $\leq$5hasFinger:
  - =5hasFinger

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Basic Facts vs. Complex Facts

- Basic facts can be represented by class assertions and/or property assertions with atomic classes and/or properties
  - Example: Obama's favorite classic is Moby-Dick by Herman Melville
    - favorite-classic (Obama, Moby-Dick)
    - author (Moby-Dick, Herman-Melville)
- Complex facts go beyond, requiring
  - Either class/property descriptions
  - Or terminological facts (also known as schema axioms)

Complex Fact Pattern (1)

- Pattern 1:
  - Entity : Class Descriptions
  - Example: Obama studied in a good school in Hawaii
    - Attempt with basic facts
      - study(Obama,school-1)
      - locatedIn(school-1,Hawaii)
      - Good(school-1)
    - Using complex pattern 1
      - Obama : ∃study.(Good ⊓ School ⊓ ∃locatedin.{Hawaii})

Complex Fact Pattern (2)

- Pattern 2:
  - Terminological fact with an enumerated class
  - Example: Obama's speeches are popular in US
    - presented(Obama,speeches)
    - popularIn(speeches,US)
  - Using complex pattern 2
    - Speech ⊓ ∃presentedby.{Obama} ⊑ ∃popularIn.{US}

Exercise: Complex Facts

- Some rich men love Jane
  - $\exists y[Rich(y) \wedge Man(y) \wedge love(y,Jane)]$
  - Jane : $\exists love^{-}.(Rich \sqcap Man)$

Exercise: Complex Facts

- All rich men love Jane
  - $\forall x[Rich(x) \wedge Man(x) \rightarrow love(x,Jane)]$
  - $Rich \sqcap Man \sqsubseteq \exists love.\{Jane\}$

Exercise: Complex Facts

- All women (except Jane) love John
  - $\forall x.[Woman(x) \wedge x \neq Jane \rightarrow love(x,John)]$
  - $Woman \sqcap \neg\{Jane\} \sqsubseteq \exists love.\{John\}$

Exercise: Complex Facts

- Jane loves both John and Jim
  - love(Jane,John)∧ love(Jane,Jim)
  - Jane: ∃love.({John} ⊔ {Jim})
  - or simply (Jane,John):love, (Jane,Jim):love

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Exercise: Complex Facts

- Jane loves either John or Jim
  - love(Jane,John)∨ love(Jane,Jim)
  - Jane: ∃love.({John} ⊔ {Jim})??
  - Jane: ∃love.({John} ⊔ {Jim}) ⊓ =1love

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Exercise: Complex Facts

- Some adult blackmails John
  - $\exists$y.[Adult(y) $\wedge$ blackmail(y,John)]
  - John: $\exists$blackmail^{-}.Adult

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Exercise: Complex Facts

- Lawers include and only include L1, ..., Ln
  - $\forall$x.[Lawer(x) -> x=L1$\vee$ ... $\vee$ x=Ln]
  - Lawer $\sqsubseteq$ {L1,...Ln}

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Exercise: Complex Facts

- Married couples include and only include (m1,w1), ..., (mn,wn)
  - $\forall$x,y.[married-couple(x,y) -> (x=m1$\wedge$y=w1)$\vee$ ... $\vee$ (x=mn$\wedge$y=wn)]
  - $\exists$married-couple.{w1} $\sqsubseteq$ {m1},...

    $\exists$married-couple^{-} {m1} $\sqsubseteq$ {w1}, ...

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Lecture Outline

- Motivation: Description Logics is the chosen family of logics for KR
- Focus: Basic Facts and Complex Facts
- Languages: RDF and OWL
- Exercises (Later on, we will explore how to construct knowledge graphs)
  - "Spain keeper Iker Casillas equalled Edwin van der Sar's record of nine clean sheets."
  - "*Six players from Barcelona were in the Spanish starting line-up*"

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