### Tutorial 3: Reasoning with Description Logics

#### Week 8

1. Check the consistency of the following knowledge graph: $K=\langle T, A \rangle$ where:
   - $T = \{ \text{Cat} \sqsubseteq \neg \text{Person}, \text{Father} \sqsubseteq \text{Person} \}$
   - $A = \{ \text{Tom} : \text{Cat} \sqcap \text{Happy} \sqcap \text{Father} \}$

2. Given the following schema $T$ of a knowledge graph $K$:
   - $T = \{ \text{Father} \sqsubseteq \text{Person}, \text{Happy} \sqcap \text{Person} \sqsubseteq \text{HappyPerson} \}$

   Check if the following subsumption holds: $\text{Happy} \sqcap \text{Father} \sqsubseteq \text{HappyPerson}$

3. Given the following schema $T$ of a knowledge graph $K$:
   - $T = \{ \text{Father} \sqsubseteq \text{Person}, \text{HappyPerson} \sqsubseteq \text{Happy} \sqcap \exists \text{hasFather.HappyPerson} \}$

   Check the satisfiability of the following concept description: $\text{HappyPerson}$

4. Consider the following TBox and ABox:

   $$ T := \{ \neg (A \sqcup B) \sqsubseteq \bot, \ A \sqsubseteq \neg B \sqcap \exists r.B, \ D \sqsubseteq \forall r.A, \ B \sqsubseteq \neg A \sqcap \exists r.A \}, $$

   $$ A := \{ r(a,b), \ r(a,c), \ r(a,d), \ r(d,c), \ (B \forall r.D)(a), \ E(b), \ (\neg A)(c), \ (\exists s.\neg D)(d) \}, $$

   Check (1) the consistency of the TBox, (2) the consistency of the ABox, (3) the consistency of the TBox and the ABox.

5. Given the following knowledge graph: $K=\langle T, A \rangle$ where:
   - $T = \{ (1) \text{AlpineClubM} \sqcap \neg \text{Skier} \sqsubseteq \text{Mountainclimber}, (2) \text{MountainClimber} \sqsubseteq \neg \exists \text{like.Rain}, (3) \neg \exists \text{like.Snow} \sqsubseteq \neg \text{Skier}, (4) \exists \text{like.}\{\text{Mike}\} \equiv \neg \exists \text{like.}\{\text{Tony}\}, (5) \text{Rain} \sqsubseteq \exists \text{like.}\{\text{Tony}\}, (6) \text{Snow} \sqsubseteq \exists \text{like.}\{\text{Tony}\} \}$
   - $A = \{ \text{Tony: AlpineClubM}, \text{Mike: AlpineClubM}, \text{John: AlpineClubM} \}$

   Check if the above knowledge graph entails $\text{Mike:MountainClimer}$.

In order to support the above knowledge graph, one might need to use some of the following extra expansion rules:

- S1-rule: if $x:N$, and $N \sqsubseteq \exists r.\{y\}$ in the TBox, then add $(x,y):r$ into the ABox;
- S2-rule: if $x:N$, and $N \sqsubseteq \exists r.\{y\}$ in the TBox, then add $(y,x):r$ into the ABox;
- S3-rule: if $x:N$, and $N \sqsubseteq \neg \exists r.\{y\}$ in the TBox, then add $(x,y):\neg r$ into the ABox;
• S4-rule: if x:N, and N ⊑ ¬∃r .{y} in the TBox, then add (y,x): ¬r into the ABox;
• S5-rule: if (y1,x): r1, and ∃r1 .{y1} ⊑ ¬∃r2 .{y2} in the TBox, then add (y2,x): ¬r2 into the ABox.