Methods for Causal Inference
Lecture 5: Rubin’s framework, propensity score, IPTW

Ava Khamseh

School of Informatics
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Potential Outcomes: Assumptions

- **SUTVA:** Stable Unit Treatment Value Assumption
  - **Consistency:** Well-defined treatment (no different versions) potential outcome is independent of how the treatment is assigned
  - **No interference**: Different individuals (units) within a population do not influence each other (e.g. does not work in social behavioural studies, care must be taken for time series data when defining the units)

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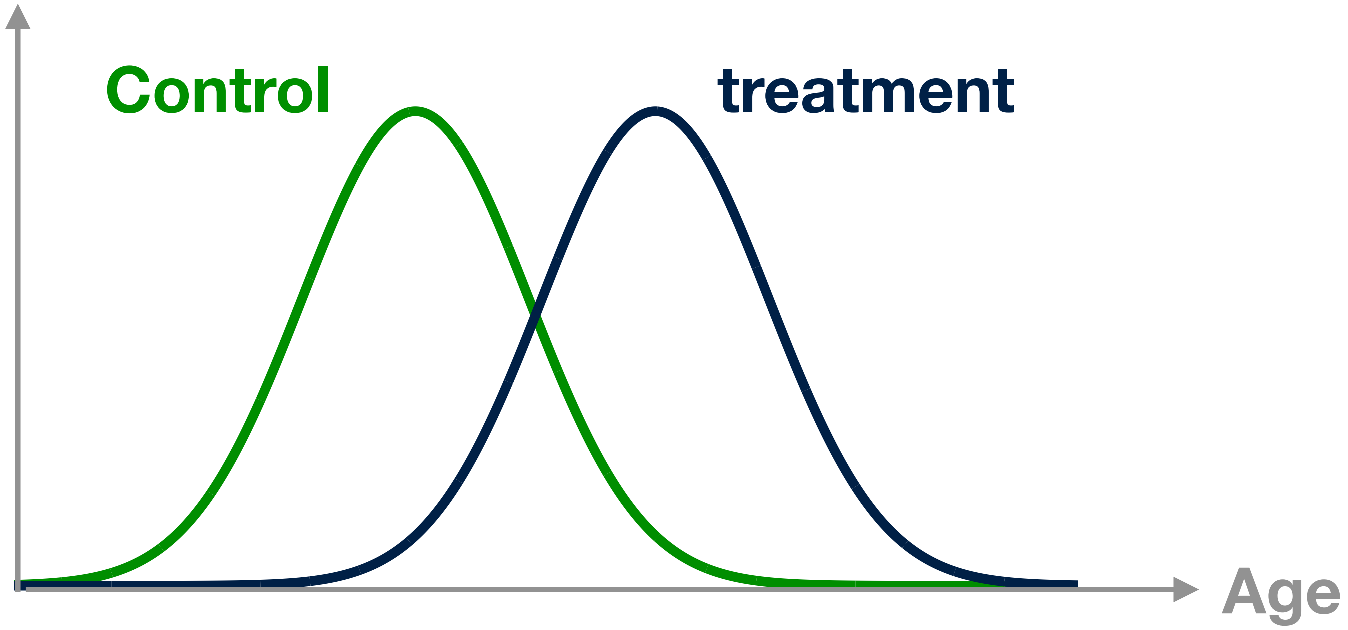
- **Positivity:** Every individual has a non-zero chance of receiving the treatment/control:

$$p(t = 1|x) \in (0, 1) \text{ if } P(x) > 0$$

- **Unconfoundedness (ignorability/exchangeability):** Treatment assignment is random, given confounding features X

Observational data: What goes wrong?

$$p(x|t=1) \neq p(x|t=0)$$



$$\left(\int y_1(x)p(x|t=1)dx \;-\int y_0(x)p(x|t=0)dx\right) \neq \int \big(y_1(x)-y_0(x)\big)p(x)dx$$

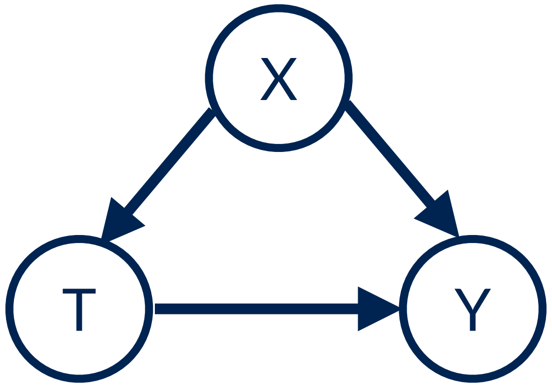
Adjustment formula (will be revisited later)

$$\begin{aligned}\mathbb{E}[Y_1 - Y_0|X] =& \mathbb{E}[Y_1|X] - \mathbb{E}[Y_0|X] \\ =& \mathbb{E}[Y_1|T=1, X] - \mathbb{E}[Y_0|T=0, X] \\ =& \mathbb{E}[Y|T=1, X] - \mathbb{E}[Y|T=0, X]\end{aligned}$$

By Unconfoundedness: $Y_1, Y_0 \perp\!\!\!\perp T \mid X$

By construction: $Y = TY_1 + (1-T)Y_0$

Also need positivity



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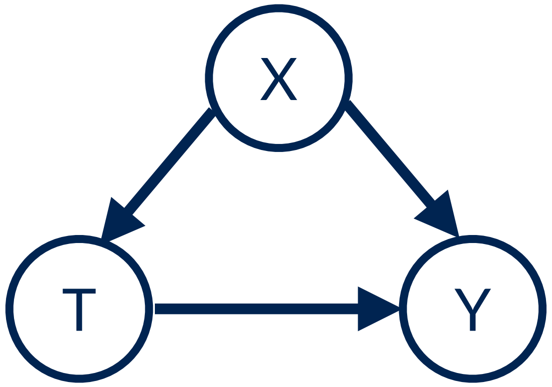
$$\mathbb{E}[Y_1 - Y_0] = \mathbb{E}_X\Big[\mathbb{E}[Y_1 - Y_0|X]\Big]$$

ATE

$$= \mathbb{E}_X\Big[\mathbb{E}[Y|T=1,X] - \mathbb{E}[Y|T=0,X]\Big]$$

The adjustment formula



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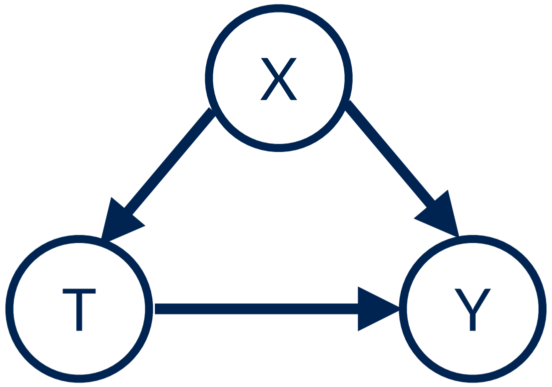
The adjustment formula

Hypothetical world

Real world

i.e., can be estimated from observational data

Causal identifiability



Regression Adjustment: Another perspective

Fit a model for $Q(T, X) = \mathbb{E}[Y|T, X]$

(last time we substituted T=1 and T=0 into individual treatment effect = $Q(1, x^{(i)}) - Q(0, x^{(i)})$, then took average over all individuals i, via linear regression). Under the linearity assumption:

$$\mathbb{E}[Y|T, X] = \alpha_0 + \beta_x X + \beta_t T + \epsilon \ , \ \mathbb{E}[\epsilon] = 0$$

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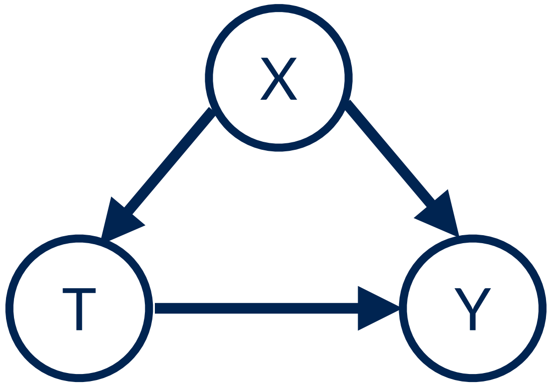
$$\begin{aligned} ATE &= \mathbb{E}_X\Big[\mathbb{E}[Y|T=1, X] - \mathbb{E}[Y|T=0, X]\Big] \\ &= \Big(\cancel{\alpha_0 + \beta_x \mathbb{E}[X]} + \beta_t\Big) - \Big(\cancel{\alpha_0 + \beta_x \mathbb{E}[X]}\Big) \\ &= \beta_t \end{aligned}$$

Important remarks about the previous form:

1) Depends on the structure of the causal graph of interest

2) Data need not be linear
   model-misspecification -> statistical bias



Important remarks about the previous form:

2) Data need not be linear, example:

Say we fitted $\mathbb{E}[Y|T,X] = \alpha_0 + \beta_x X + \beta_t T + \epsilon \ , \ \mathbb{E}[\epsilon] = 0$
And obtained β_t for the causal effect,

BUT, in reality the true data generating distribution is e.g.

$$\mathbb{E}[Y|T,X] = \alpha_0 + \beta_x X + \beta_t T + \gamma X.T + \epsilon \ , \ \mathbb{E}[\epsilon] = 0$$

Or e.g. non-linear:

$$\mathbb{E}[Y|T,X] = e^{\alpha_0 + \beta_x X + \beta_t T + \gamma X.T}$$

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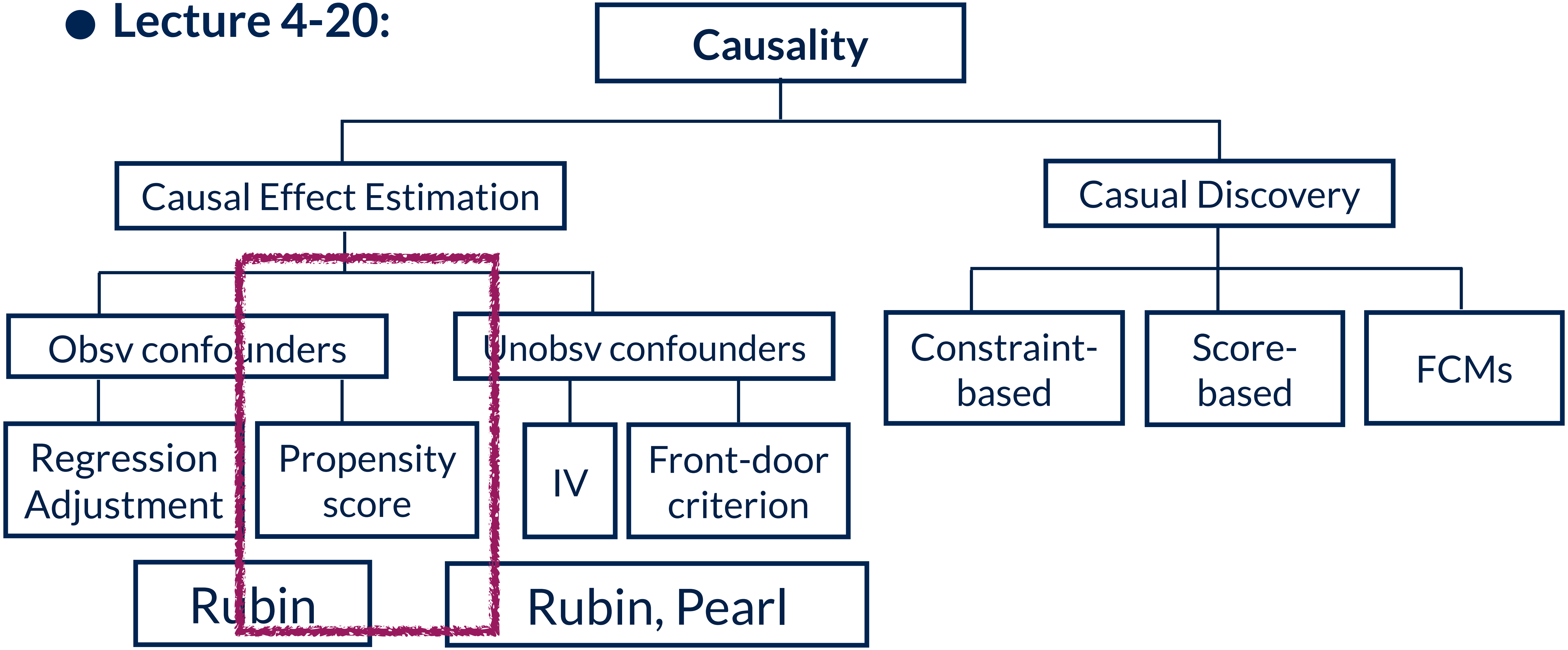
Then $ATE = \mathbb{E}_X \Big[\mathbb{E}[Y|T=1,X] - \mathbb{E}[Y|T=0,X] \Big]$

is **not** simply β_t !!

Valid causal inference requires correctly-specified models and mathematical guarantees!

Overview of the course

- **Lecture 1**: Introduction & Motivation, why do we care about causality? Why deriving causality from observational data is non-trivial.
- **Lecture 2:** Recap of probability theory, variables, events, conditional probabilities, independence, law of total probability, Bayes' rule
- **Lecture 3**: Recap of regression, multiple regression, graphs, SCM
- **Lecture 4-20:**

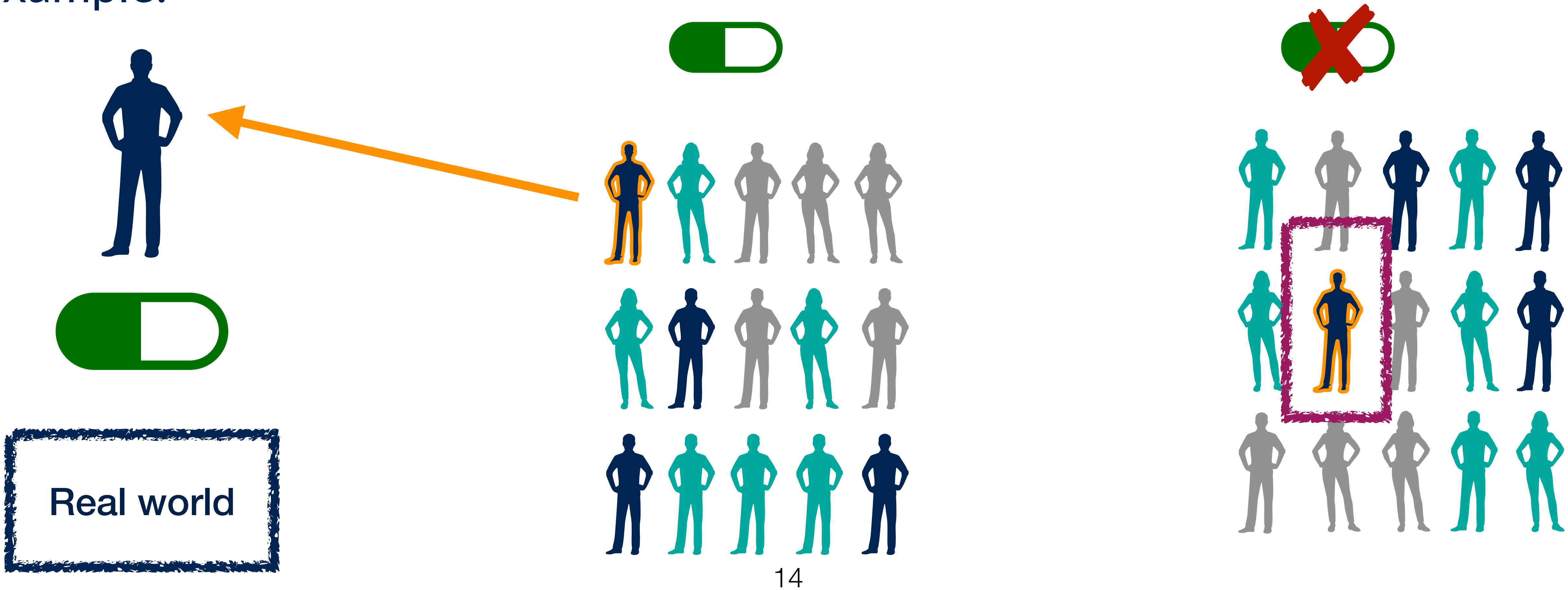


Matching

Idea: Create a 'clone/twin' for each individual (in terms of X)
i.e. if individual 1 has t = 1, then their 'clone/twin' has t = 0.

Blind ourselves to the outcomes, try to get as similar to a randomised experiment as possible ('correct for confounding')

Example:



Real world

Balancing Score

- In a perfect **randomised** trial: p(t=1|x)=p(t=1)

- In an **observational study**, p(t=1|x) can be **estimated**, since it involves **observational data** at a t and x (hence identifiable).

- A **balancing score** is any function b(x) such that:

$$x \perp\!\!\!\perp t | b(x)$$

- i.e., distribution of confounders is independent of treatment given b(x):

$$p(X = x | b(x), t = 1) = p(X = x | b(x), t = 0)$$

Rosenbaum, Rubin 1983

Balancing Score: Proof 1 [non-examinable]

Unconfoundednesss given a balancing score. Suppose we have unconfoudedness, i.e., $Y_1^{(i)}, Y_0^{(i)} \perp\!\!\!\perp T^{(i)} \mid X^{(i)}$. Then for a balancing score b(x) we have:

$$Y_1^{(i)}, Y_0^{(i)} \perp\!\!\!\perp T^{(i)} \mid b(X^{(i)})$$

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$$\mathbb{E}_W[\mathbb{E}_{Z|W}[Z|W]] = \sum_w \sum_z p(Z = z | W = w) \; z \; p(W = w)$$

$$= \sum_{w,z} \frac{p(z, w)}{p(w)} \; z \; p(w) = \sum_z p(z) \; z = \mathbb{E}_Z[Z]$$

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$$= \mathbb{E}_{X|Y_1,Y_0,b(X)}\Bigg[\mathbb{E}\Big[T | Y_1, Y_0, b(X), X\Big] | Y_1, Y_0, b(X)\Bigg]$$

$$\boxed{\mathbb{E}[Z|W] = \mathbb{E}_{V|W}[\mathbb{E}[Z|W,V]|W]}$$

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Casual Inference by Imbens and Rubin

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By definition of balancing score: $X \perp\!\!\!\perp T|b(X)$

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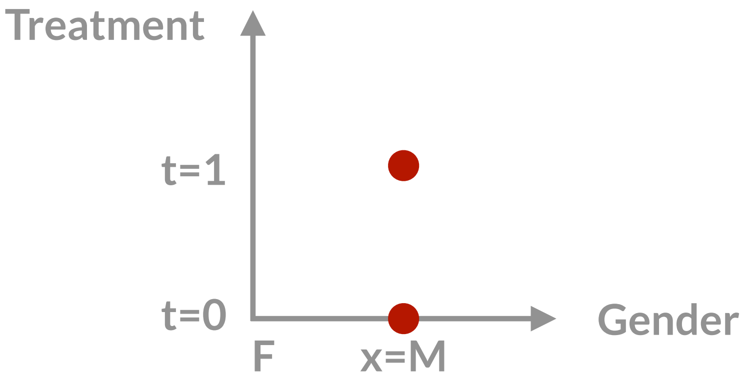
Casual Inference by Imbens and Rubin

Propensity Score

- Candidate b(x) = x, trivially satisfies:

$$p(X = x|x, t = 1) = p(X = x|x, t = 0) = 1$$

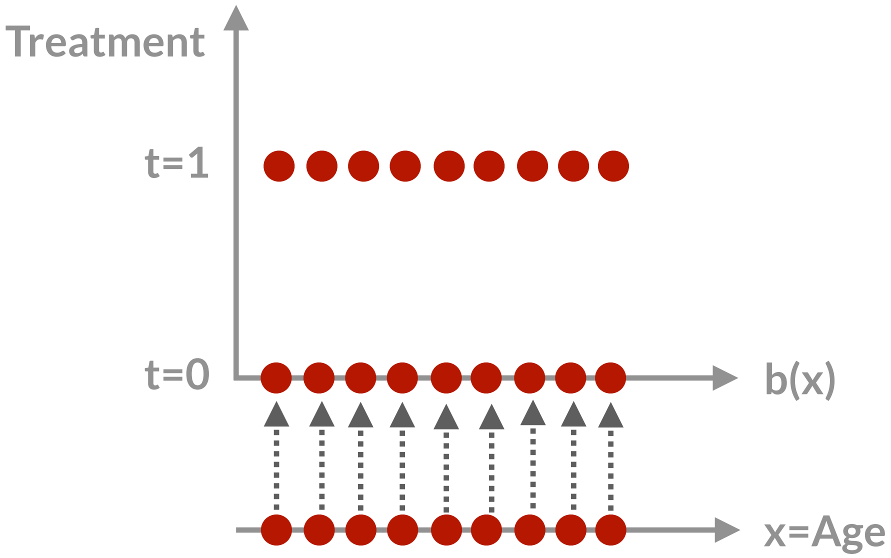


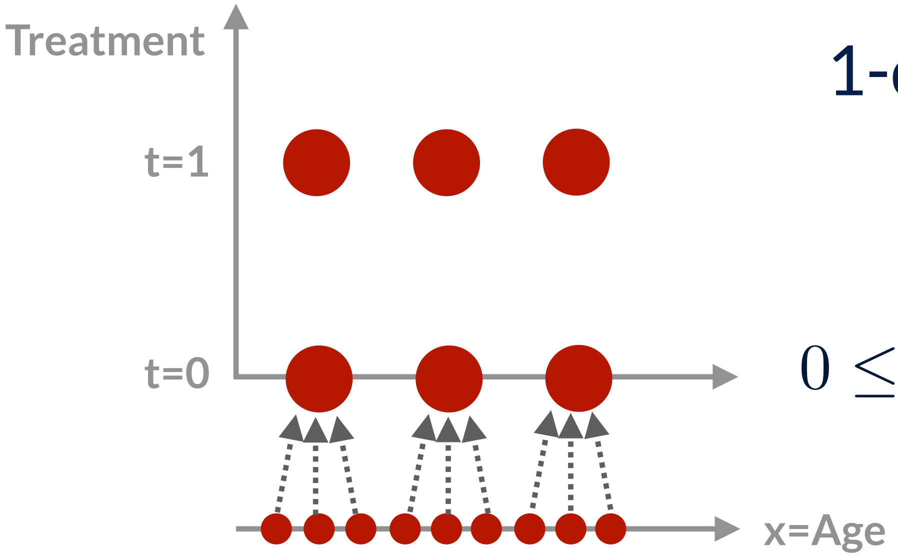
- b(x) = x is the **finest** such function: OK for e.g. binary confounders, but only gives point estimates for (almost) continuous confounders!

- **Propensity score** is the **coarsest** such function (i.e. more data points, leading to better estimates):

$$\boxed{e(x) = p(t = 1|x)}$$

1-dimensional





$$0 \leq e(x) \leq 1$$

Propensity is balancing: Proof 2 [non-examinable]

The propensity score is a balancing score: $X \perp\!\!\!\perp T | e(X)$

Proof: Need to show $p_T\Big(T = 1 | X, e(X)\Big) = p_T\Big(T = 1 | e(X)\Big)$

LHS: $p_T\Big(T = 1 | X, e(X)\Big) \underset{\substack{\big\uparrow \\ \text{Propensity score} \\ \text{is a function of X}}}{=} p_T\Big(T = 1 | X\Big) \underset{\substack{\big\uparrow \\ \text{Propensity score} \\ \text{definition}}}{=} e(X)$

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$$= \mathbb{E}[e(X)|e(X)] = e(X)$$

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The propensity score is the coarsest balancing score, i.e., it is a function of every balancing score $b(x)$: $e(x) = f(b(x))$

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$$p(t=1|x, \underline{b(x)}) = p(t=1|x) = e(x) \boxed{\neq} e(x') = p(t=1|x') = p(t=1|x', \underline{b(x')})$$

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$$p(t = 1|x, \underset{b^*}{b(\cancel{x})}) \neq p(t = 1|x', \underset{b^*}{b(\cancel{x'})})$$

Recall definition of balancing score: $x \perp\!\!\!\perp t|b(x)$

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$$p(t = 1|x, b(x)) \enclose{circle}{\neq} p(t = 1|x', b(x'))$$

i.e., probability of treatment changes depending on value of x **despite b***:

$$x \not\perp\!\!\!\perp t|b(x)$$

This violates the definition of a balancing score.
Proof by contradiction.

Propensity Score Matching Algorithms

- Match control and treatment individuals based on their propensity score
- Greedy matching:
  - Randomly order list of control and treated.
  - Start with the first individual from e.g. treated and match to control with the smallest distance (i.e. obtains the **local** minimum)
  - Remove individuals from control and matched treated
  - Move to the next treated subject

Treatment	Control
40	50
65	25

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Treatment Control

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Total diff: 50

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- Optimal matching: Minimises the **global** distance, computationally demanding

- **ATE:** $\tau = \hat{\mathbb{E}}[\tau^{(i)}] = \hat{\mathbb{E}}[y_1^{(i)} - y_0^{(i)}] = \frac{1}{N}\sum_{i=0}^{N}\left(y_1^{(i)} - y_0^{(i)}\right)$

Inverse Probability of Treatment Weighting (IPTW)

- Inflate the weight for under represented-subjects due to missing data
- Based on propensity score
- Weight: inverse probability of receiving observed treatment, for individual i with covariate x:

$$w_i = \begin{cases} \frac{1}{e(x_i)} & \text{if } t_i = 1 \\ \frac{1}{1-e(x_i)} & \text{if } t_i = 0 \end{cases}$$

$$\boxed{e(x) = p(t = 1|x)}$$

- Example: Suppose individual (i) has a large e(x), i.e., their probability of receiving treatment is high.

- If $t_i = 1$ then $w_i \approx 1$ (typical behaviour: most with x_i are treated)
- If $t_i = 0$ then $w_i \gg 1$ (underrepresented: boost weight for rare event)

Rosenbaum 1987

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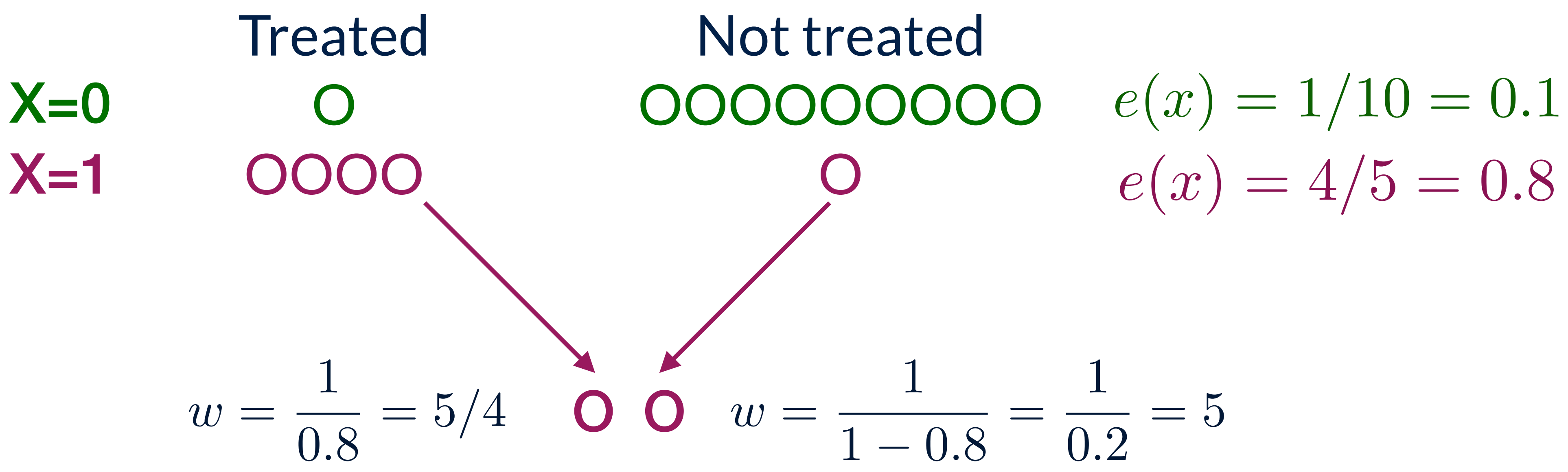
- Inflate the weight for under represented-subjects due to missing data
- Based on propensity score
- Weight: inverse probability of receiving observed treatment, for individual i with covariate x:

$$w_i = \begin{cases} \frac{1}{e(x_i)} & \text{if } t_i = 1 \\ \frac{1}{1-e(x_i)} & \text{if } t_i = 0 \end{cases}$$

$$\boxed{e(x) = p(t = 1|x)}$$



	Treated	Not treated	
X=0	O	OOOOOOOOO	$e(x) = 1/10 = 0.1$
X=1	OOOO	O	$e(x) = 4/5 = 0.8$

$w = \frac{1}{0.8} = 5/4$ O O $w = \frac{1}{1-0.8} = \frac{1}{0.2} = 5$

Rosenbaum 1987

Inverse Probability of Treatment Weighting (IPTW)

$$\boxed{e(x) = p(t=1|x)}$$

$$\frac{1}{N}\sum_{\text{treated}} y_{\underline{1}}^{(i)} \frac{1}{e(x_i)} - \frac{1}{N}\sum_{\text{not treated}} y_{\underline{0}}^{(i)} \frac{1}{1-e(x_i)}$$

Weights may be inaccurate/unstable for subjects with a very low probability of receiving the observed treatment (other estimators exist)

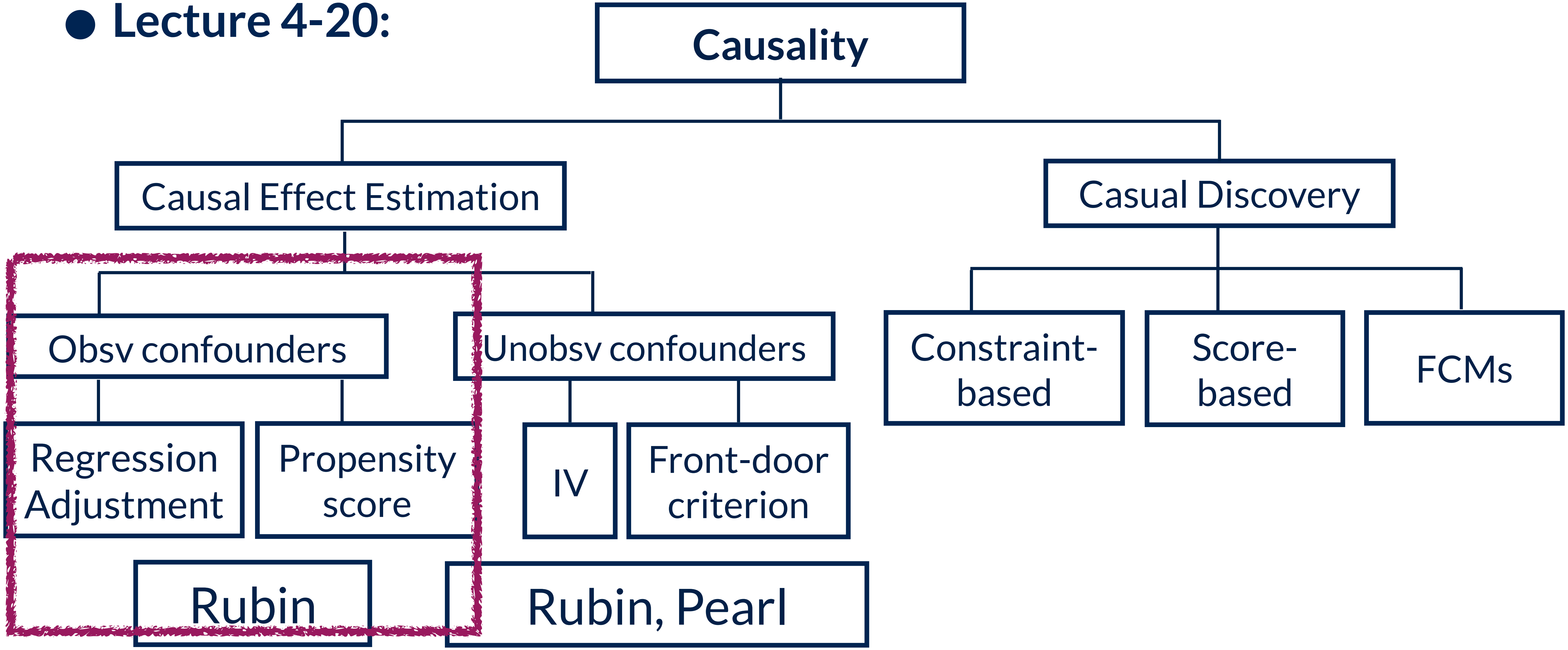
In a randomised control trial (RCT) limit, $p(t=1|x) = p(t=0|x)$ above reduces to:

$$N = N_1 + N_0$$

$$\frac{1}{N_1}\sum_{\text{treated}} y_{\underline{1}}^{(i)} - \frac{1}{N_0}\sum_{\text{not treated}} y_{\underline{0}}^{(i)}$$

Overview of the course

- **Lecture 1**: Introduction & Motivation, why do we care about causality? Why deriving causality from observational data is non-trivial.
- **Lecture 2:** Recap of probability theory, variables, events, conditional probabilities, independence, law of total probability, Bayes' rule
- **Lecture 3**: Recap of regression, multiple regression, graphs, SCM
- **Lecture 4-20:**



Methods for Causal Inference
Lecture 5: Rubin’s framework, propensity score, IPTW

Ava Khamseh

School of Informatics
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