Privacy Preserving Machine Learning

Machine Learning Theory (MLT)

Edinburgh

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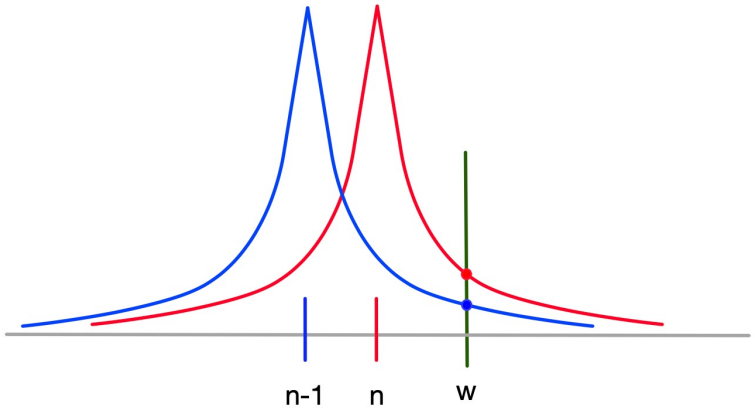
Coursework

- Keep an eye on piazza
- Note that submission deadline is 13 march at 12:00.
  - Submit early. Don’t keep till last minute!

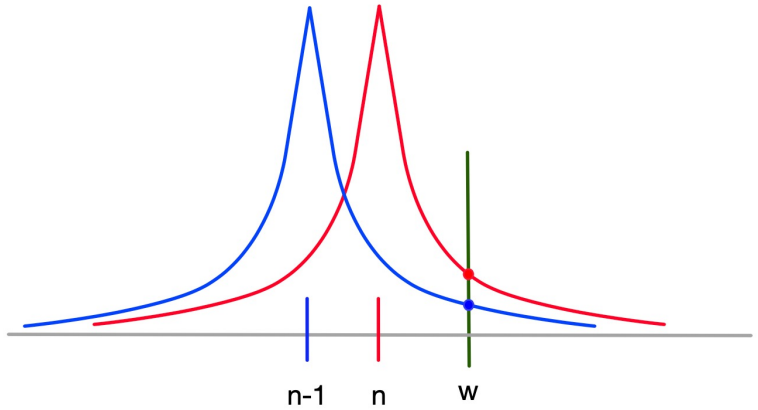
Recap: DP example: Make output probabilities similar for n and n-1

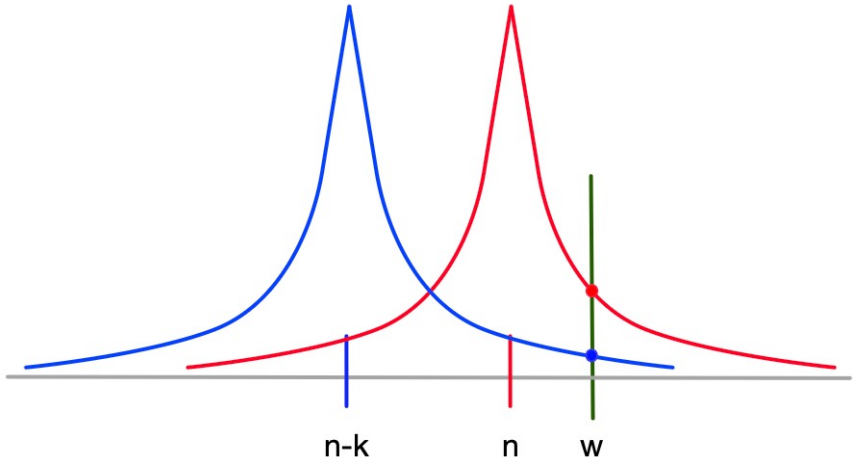
- Problem: count the number of people in the meeting
- Solution:
  - Find real count n
  - Publish $n + y$ where y is noise (a random number)
    - Draw y from Laplace distribution
    - $\Pr[y] = \frac{1}{2b} e^{-\frac{|y-\mu|}{b}}$
    - μ is the mean, $2b^2$ is variance
- We will write $Lap(b)$ to mean Laplace distribution with mean 0 and variance $2b^2$



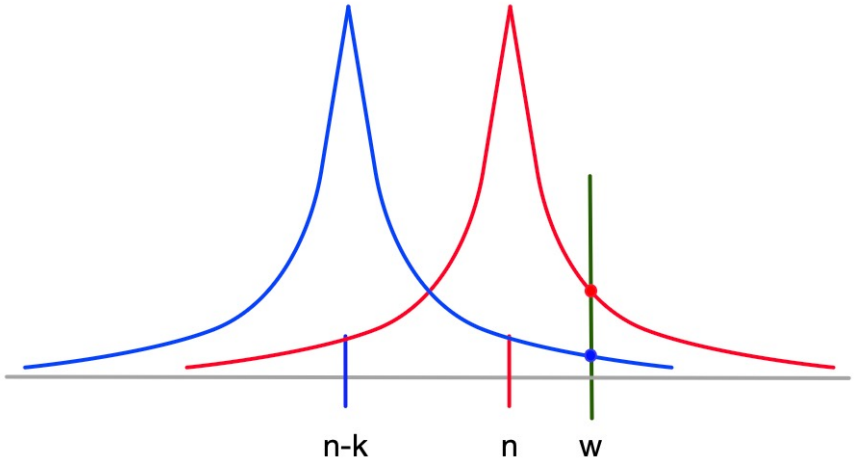
What about privacy other than one person?

- What about a group of k?
- Suppose there is a group of k people who may not have attended.
  - Neighboring databases differ in count of k
  - A more general notion of neighboring databases
- And we want to hide this from the observer/adversary
  - Should have similar probabilities for n and n-k
- Exercise : Show that an ϵ-DP algorithm is $k\epsilon$-DP for any group of size k

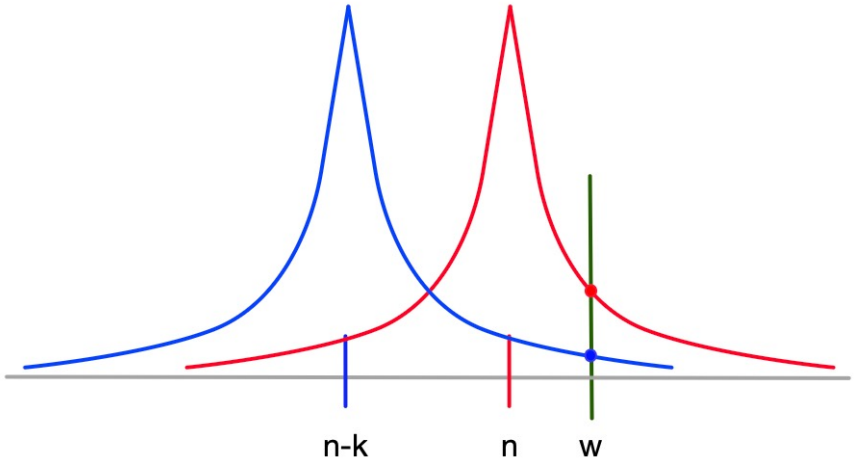




- Any published w can occur in two ways:
- $n + y$
- $(n - k) + (y + k)$
- The ratio $\frac{\Pr[noise=y]}{\Pr[noise=y+k]} =?$



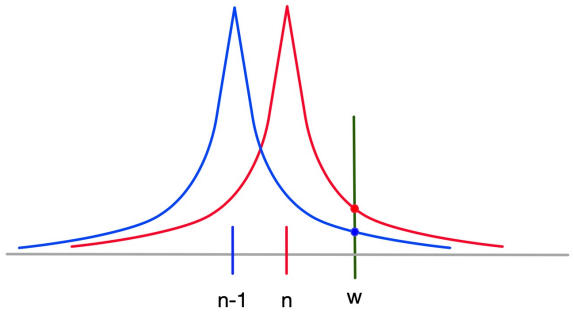
- Any published w can occur in two ways:
- $n + y$
- $(n - k) + (y + k)$
- The ratio $\frac{\Pr[noise=y]}{\Pr[noise=y+k]} = \frac{e^{-|y|\epsilon}}{e^{-|y+k|\epsilon}} \leq e^{k\epsilon}$

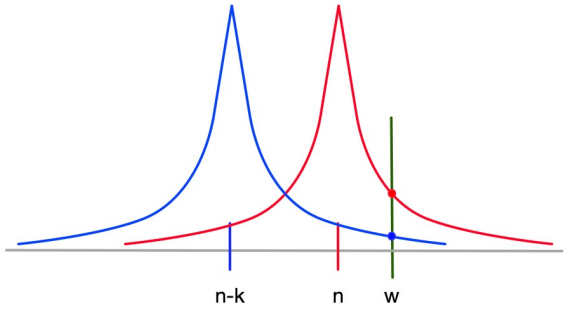


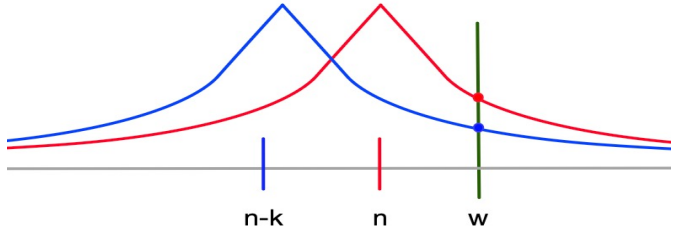
- Remember that $k\epsilon$ is worse privacy than ϵ
  - Larger groups are harder to privatise!
  - How can we get ϵ-DP for k people?

ϵ-DP for a group of k-people

- Add a noise of $Lap\left(\frac{k}{\epsilon}\right)$
- The larger variance makes the red and blue distributions more similar
  - Similar output probabilities
- Attains ϵ-DP
- Observe that it works to hide any group of k people not attending







How much noise do we need?

- The *variance* of the noise depends on how much f can change due to a unit change between neighboring databases
- If $f(D)$ and $f(D')$ are every close, we need only a little noise to make them indistinguishable
  - If they are very different, we need a large noise

- *Sensitivity* of f is written as Δf :
  - $\Delta f = \max_{D,D'} |f(D) - f(D')|$
  - The maximum possible difference in f between neighboring datasets.

Laplace mechanism

- Given a database D and a function f,
- Output $A(D) = f(D) + y$, where $y \sim Lap\left(\frac{\Delta f}{\epsilon}\right)$

- This ensures ϵ differential privacy.

Privacy and utility

- Adding noise gains privacy
- Adding more noise (from a distribution of higher variance) gains more privacy
- But more noise decreases accuracy of the eventual use of the value, so it is less useful
  - We say it has less *utility*
- The challenge is to balance utility and privacy
  - By adding the minimum necessary noise
- If sensitivity is larger, noise is larger. Utility is less.

Examples

- When f is the count of number of people
  - $\Delta f = 1$
- Suppose the max height of a person is assumed to be 7ft.
  - If f is the sum of heights of a group in feet, then sensitivity is 7.

- Suppose the max salary in a company is capped at £50K
  - We are analysing total salaries in groups of size 4. That is, a group is either present or absent. Then the sensitivity is £200K.

Another example

- Suppose $f = \frac{1}{n}\sum X_i$, where $X_i \in [0,1]$
- Sensitivity is $\frac{1}{n}$ (if we replace one number by another)
  - Note that we are using the replacement version for convenience.
- Thus, the noise required is $y \sim Lap\left(\frac{1}{\epsilon n}\right)$

Utility of Laplace mechanism

- We add noise $y \sim Lap\left(\frac{\Delta f}{\epsilon}\right)$
- But noise hurts the utility.
  - Is the output of Laplace mechanism useful?

- We can show that the error from Laplace mechanism is limited
- Laplace method has an error bound (utility guarantee):
  - Expected error $\mathbb{E}[|A(D) - f(D)|] = \frac{\Delta f}{\epsilon}$
  - So the error induced is determined by sensitivity, but is not much larger

Local differential privacy

- Suppose $X_1, X_2, \ldots$ are in [0, 1] and represent how frequently person i uses their phone
- Suppose they are recorded by mobile phones of person $1,2,\ldots$ and transmitted to a server
- The problem: Each person wants their data to be private
  - E.g. when they do not trust the server collecting and computing
- Solution:
  - Add noise *before* transmitting the variables
  - Ie. Each phone i Transmits $X_i + y_i$, where $y_i \sim Lap\left(\frac{1}{\epsilon}\right)$
  - Either the server, or anyone snooping the line cannot be sure of the data transmitted
  - Data is protected by DP right from the source.
  - Disadvantage: A lot more noise than adding noise once

Approximate differential privacy

- Algorithm A is (ϵ, δ)-differentially private if :
  - For every neighbouring D, D'
  - $\Pr[A(D) \in S] \leq e^{\epsilon} \cdot \Pr[A(D') \in S] + \delta$

- In theory, δ must be very small to be useful, like $o\left(\frac{1}{n}\right)$.
  - In practice, it is sometimes a bit larger.

- When $\delta = 0$, we have pure DP (the version we saw already)

Gaussian mechanism

- Gaussian distribution: $\mathcal{N}(\mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{\left(-\frac{(y-\mu)^2}{2\sigma^2}\right)}$
- Compute $f(D)$ and Δf
- Draw noise from Gaussian distribution $y \sim \mathcal{N}(0, \sigma^2)$
  - Where $\sigma = \frac{\left(\sqrt{2\ln\left(\frac{1.25}{\delta}\right)} \cdot \Delta f\right)}{\epsilon}$
- Output $f(D) + y$
- Theorem: Gaussian mechanism satisfies (ϵ, δ)-DP

Composition

- Suppose we run algorithms $A_1, A_2, \ldots.$
  - Where A_i is (ϵ_i, δ_i)-DP

- E.g. we perform different computations on the same data and publish the results
- Then the end result $(A_1(D), A_2(D), \ldots A_n(D))$ is
- (ϵ, δ)- DP for
  - $\epsilon = \sum \epsilon_i,$
  - $\delta = \sum \delta_i$

- Thus, ϵ, δ increase and we lose privacy as more computations are released

Repeated computation

- The same idea holds if we carry out the same operation multiple times
- E.g. if A_1, A_2, … are all the same computation and each adds a noise, then, while each may be (ϵ, δ) DP, the overall privacy loss can be large for many repetitions

With k repetitions of the same query

- How can we get ϵ-DP?

Resilience to post processing

- Differential privacy holds for further use of the output

- E.g. if a model or output m is obtained with an ϵ-DP algorithm, then we can use m as many times as we wish without any additional loss of privacy
  - We are not using the raw database D any more, and using m again does not lose more privacy
  - If a computation uses D again, then it loses more privacy.

Notes on differential privacy

- DP is a property of the randomized algorithm
- The algorithm depends on the assumptions and computation objective
  - Not on the dataset
  - Noise depends on sensitivity (from what is known/assumed about the data)
    - Note that sensitivity is not *computed* from the data, as this itself will leak privacy
    - It is assumed that whatever is needed to compute sensitivity is known to everyone
  - Noise is computed for the objective, for *all possible inputs*
  - Noise is not computed for specific input points

- DP is determined based on the assumptions about the data, but not on the actual data.
  - If your DP mechanism depends on anything computed from the data, be very careful!

Algorithms with multi-dimensional outputs

- Till now, we have considered algorithms with real valued outputs
- Now, consider algorithms with k-dimensional outputs
- E.g. ML models with k parameters

- Similar to applying k different algorithms

- Suppose $f: \mathcal{Z}^n \rightarrow \mathbb{R}^k$
  - Produces a k-dim output

- On changing the input D, all k different parameters may change
- How do we measure sensitivity?

L_p Norms

- L_p norms are defined as:
  - $|x|_p = \left(x_1^p + x_2^p + \cdots + x_k^p\right)^{1/p}$

- Most relevant for us:
  - $|x\ - y|_1 = |x_1 - y_1| + |x_2\ - y_2| + \cdots + |x_k - y_k|$
  - $|x\ - y|_2 = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \cdots + (x_k - y_k)^2}$

L_1 and L_2Sensitivity

- $f: \mathcal{Z}^n \rightarrow \mathbb{R}^k$
  - Produces a k-dim output

- Sensitivity measured as the norms:
- $\Delta_1 f = \max_{D,D'} |f(D) - f(D')|_1$

- $\Delta_2 f = \max_{D,D'} |f(D) - f(D')|_2$

k-dim Laplace mechanism

- Compute $\Delta_1 f$
- For $i = 1, \dots k$, draw $y_i \sim Lap\left(\frac{\Delta_1 f}{\epsilon}\right)$
- Output $f(D) + Y$, where $Y = (y_1, y_2, \dots y_k)$

k-dim Gaussian mechanism

- Compute $\Delta_2 f$
- For $i = 1, \dots k$, draw $y_i \sim \mathcal{N}(0, \sigma^2)$
  - Where $\sigma = \dfrac{\left(\sqrt{2 \ln\left(\frac{1.25}{\delta}\right)} \cdot \Delta_2 f\right)}{\epsilon}$
- Output $f(D) + Y$, where $Y = (y_1, y_2, \dots y_k)$

Which is better?

- Consider L1 and L2 norms for k dimensions
- Assume x – y = [1,1,1,1,1,…]
- $|x - y|_1 = k$
- $|x - y|_2 = \sqrt{k}$

- Thus, in high dimensions, L2 sensitivity is smaller.
  - Gaussian mechanism can use L2 sensitivity
- Since Sensitivity determines standard deviation of distribution, gaussian mechanism has smaller standard deviation (or variance).
- But remember that gaussian mechanism also has the additive δ

Perturbation mechanisms

- Input perturbation:
  - Modify the data (add noise to each point) before any computation is applied
  - Similar to local DP
- Output perturbation
  - Compute output, then add the noise
  - E.g. add noise to the aggregate, or to the ML model parameters
- Gradient or internal perturbation
  - Add noise to the operation of the algorithm
  - E.g. to the gradients of SGD

Differentially Private Empirical Risk Minimisation

- Regularized ERM:
  - $\widehat{\boldsymbol{w}} = argmin_{\boldsymbol{w}}(L_S(\boldsymbol{w}) + R(\boldsymbol{w}))$: Optimal regularized model
  - $R(\boldsymbol{w})$ is a regularizer
  - Common choice: $R(\boldsymbol{w}) = \lambda\left|\left|\boldsymbol{w}\right|\right|^2$ is 2λ-strongly convex
  - Makes $(L_S(\boldsymbol{w}) + R(\boldsymbol{w}))$ strongly convex

- Theorem: L_2 sensitivity for ERM:
  - If loss function L is λ-strongly convex
  - If L is convex, differentiable, G-Lipschitz
  - Then the L_2 sensitivity is at most $\frac{4G}{n\lambda}$

DP-ERM using output perturbation

- Optimal solution: $\widehat{\boldsymbol{w}} = argmin_{\boldsymbol{w}}(L_S(\boldsymbol{w}) + R(\boldsymbol{w}))$
- For $i = 1, \dots k$, draw $y_i \sim \mathcal{N}(0, \sigma^2)$
  - Where $\sigma = \frac{\left(4G\sqrt{2\ln\left(\frac{1.25}{\delta}\right)}\right)}{n\lambda\epsilon}$
- $\widehat{\boldsymbol{w}} + Y$, where $Y = (y_1, y_2, \dots y_k)$

- Apply gaussian mechanism with the right sensitivity
- Guarantees (ϵ, δ)-DP

DP-SGD

- At every step:
  - Sample a point or batch z
  - Compute Gradient $\mathrm{g_t} = \nabla f_{\boldsymbol{w_t}}(z)$ (privacy risk: gradient is a function of z)
  - clip g_t to length at most C : $\widehat{g_t} \leftarrow g_t \cdot \frac{1}{\max(1, \frac{||g_t||}{C})}$
  - Add noise $\widehat{g_t} = \widehat{g_t} + r$ where $r \sim \mathcal{N}(0, \sigma^2 C^2)$ (in suitable dimensions..)
  - Update model $\boldsymbol{w}_{t+1} = \boldsymbol{w}_t - \eta g_t$
- At end, return final model

Differential privacy of DP-SGD

- The impact of data point(s) used in any one step is limited to a vector of length C
- For $\sigma = \frac{\sqrt{2 \ln \frac{1.25}{\delta}}}{\epsilon}$, every step is (ϵ, δ)-DP
- Overall, for T steps with batches that are q fraction of data, we get $(qT\epsilon, qT\delta)$-DP
- More tighter bounds available, e.g. $(q\epsilon\sqrt{T}, \delta)$
- Proofs omitted

Amplification by sampling

- If A is an (ϵ, δ)-DP algorithm for D
- If $S(D)$ returns a sample of D where each element is present with probability p

- Then $A(S(D))$ is $(\epsilon', p\delta)$-DP, where
  - $\epsilon' = \ln(1 + p(e^{\epsilon} - 1))$
  - Note that $\ln(1 + p(e^{\epsilon} - 1)) \leq 2p\epsilon$

- Sampling improves the privacy of a private algorithm (e.g. in randomly selected batches)

DP machine learning

- Current research area, with connections to all aspects of ML
  - Complexity, optimization, stability, dimension reduction….
- For any datascience/ML algorithm it is possible to ask: What is the DP version of this algorithm? Can we get a good enough privacy – utility tradeoff?
- Current situation with DP-SGD and differentially private training
  - Works well on smaller models
  - On large neural networks, the extra noise with many parameters makes them inaccurate
  - Ongoing research in making differential privacy more useful
    - Typical approacg: argue that overall small levels of noise added carefully actually suffices to get required levels of privacy

Non-numerical queries

- Our basic output perturbation mechanism: $A(D) = f(D) + Y$
  - Works only with numeric values
- How can we privatise non-numeric queries?
  - What is the most popular movie?
  - Which date is likely to work best to schedule a meeting?
  - What are other examples of non-numeric or selection queries?
  - How can we privatize them?

Non-numerical queries

- Suppose $f: \mathcal{Z}^n \rightarrow \mathcal{O}$ (for dataset $D \in \mathcal{Z}^n$)
  - $\mathcal{O}$ is a set of discrete possibilities
    - E.g. set of movies, set of computers, set of web sites etc
- While $\mathcal{O}$ is discrete, we can imagine a numerical score or utility to elements of $\mathcal{O}$
  - E.g. the popularity of a movie, the number of people that can attend the meeting on a date etc
  - $s: \mathcal{Z}^n \times \mathcal{O} \rightarrow \mathbb{R}$
  - $s(D, o)$ represents the score of o as an approximation of $f(D)$

Sensitivity of s

- For neighbouring D, D'
- $\Delta s = \max_{o \in \mathcal{O}} \max_{D, D'} |s(D, o) - s(D', o)|$
- Worst case difference in output between neighboring datasets

Exponential mechanism

- Task is to find f(D)
- Compute s and Δs
- Output o with probability
  - $\Pr[o] = \frac{e^{\frac{s(D,o)\epsilon}{\Delta s}}}{\sum_{o' \in \mathcal{O}} e^{\frac{s(D,o')\epsilon}{\Delta s}}}$
- Sample o with probability proportional to its score
  - Denominator is just normalisation
- Probability drops off exponentially with decreasing score
- Compare with Laplace mechanism

Theorem

- Exponential mechanism satisfies ϵ-DP