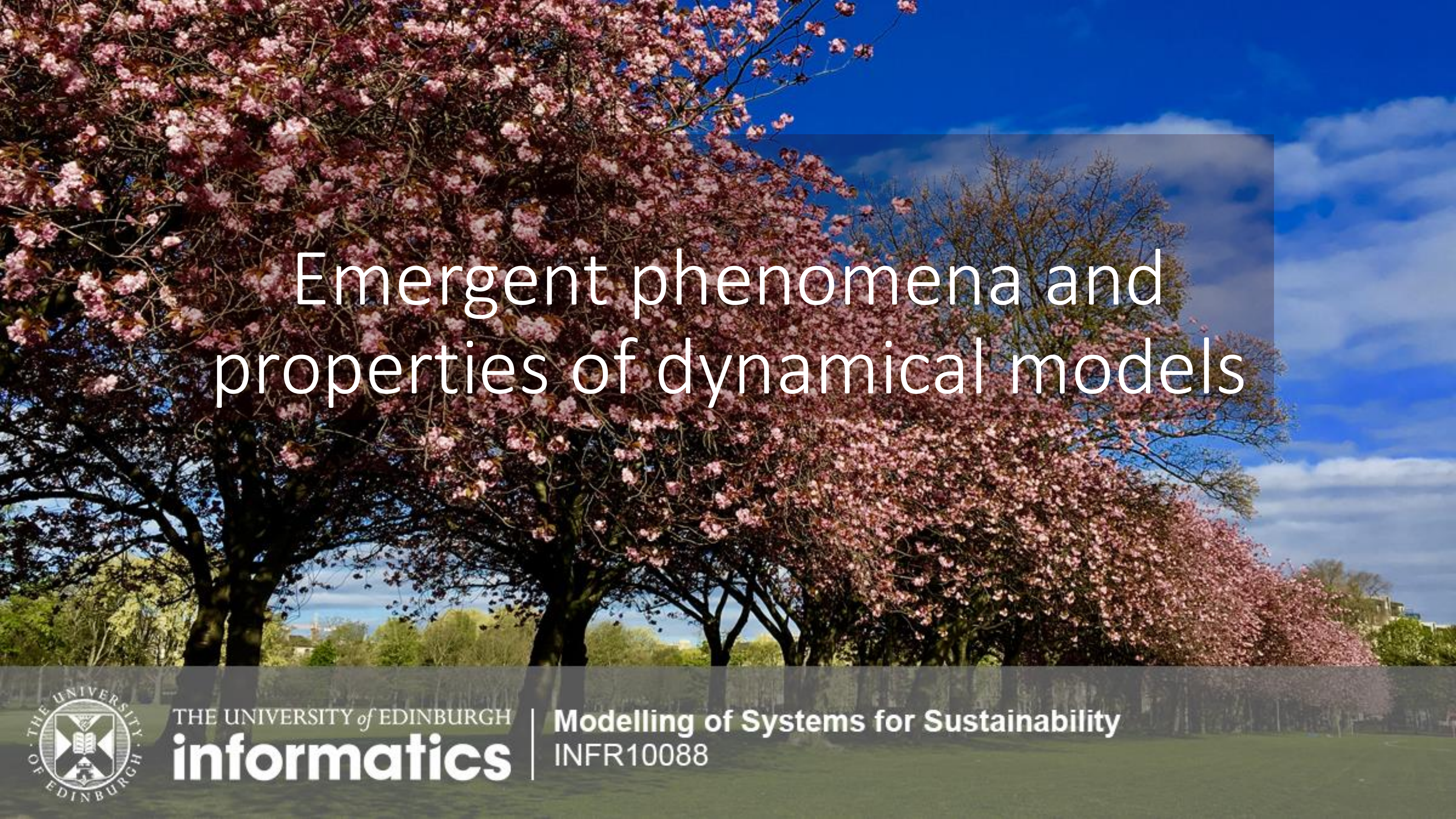
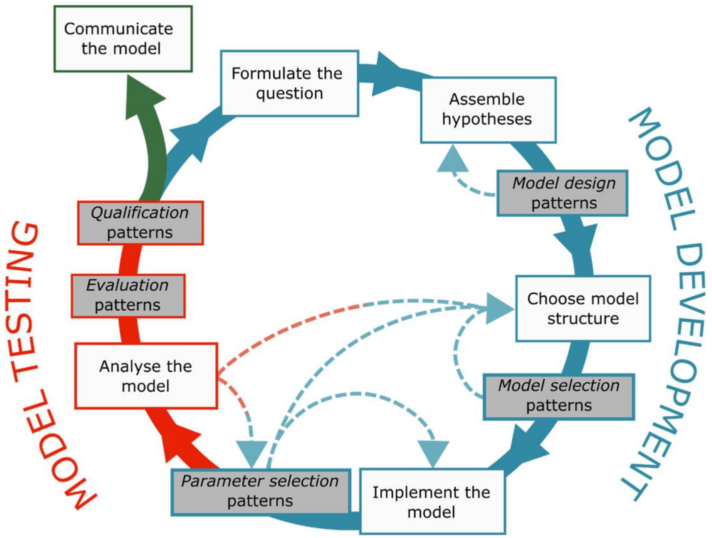


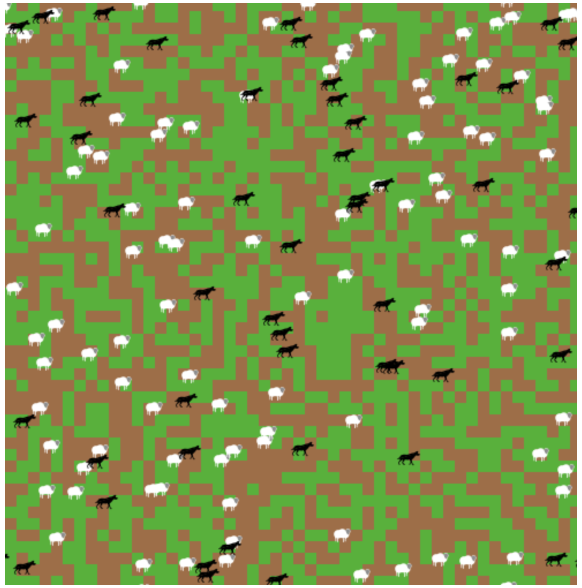
Emergent phenomena and properties of dynamical models

THE UNIVERSITY of EDINBURGH informatics | **Modelling of Systems for Sustainability**
INFR10088

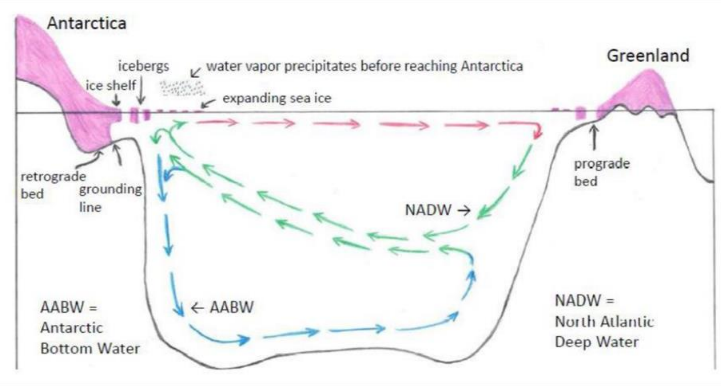
So far...



The modelling cycle

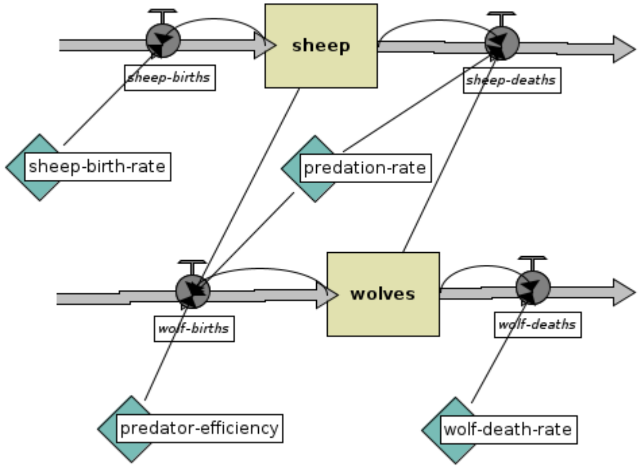


Agent-based modelling...



Real systems, e.g. Atlantic Meridional Overturning Circulation

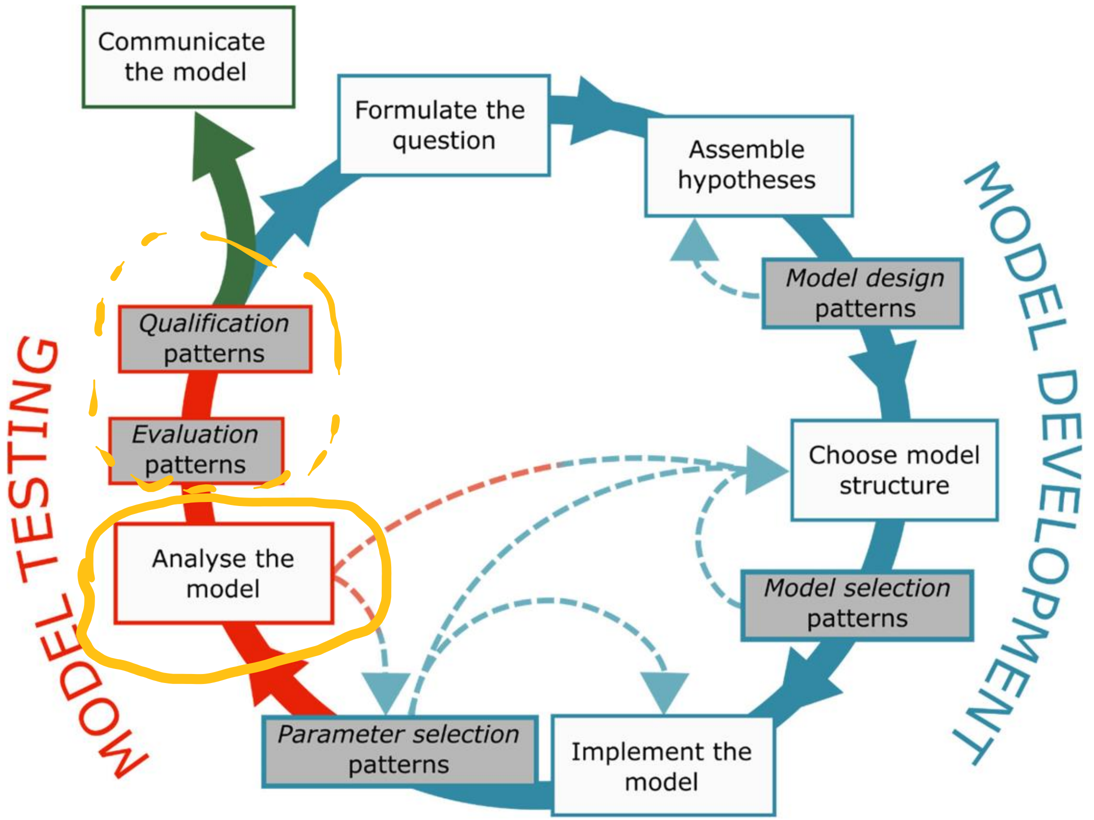
Overview
Design concepts
Details



...and system dynamics

Today...

- **D**esign concept: Emergence
- Model analysis: Properties of dynamical systems
- Some:
    - Quantification of patterns
    - Exploration of parameter space



Overview - aims

- Overall aim (Course Learning Outcome 1):
  "explain how computational modelling frameworks can be used to understand the behaviours of complex interacting systems involved in sustainability such as social, economic and ecological systems"
- This lecture:
  - Emergent Phenomena
  - Properties of dynamical models
  - Example system

What do we mean by "emergent phenomena"?

What do we mean by emergence?

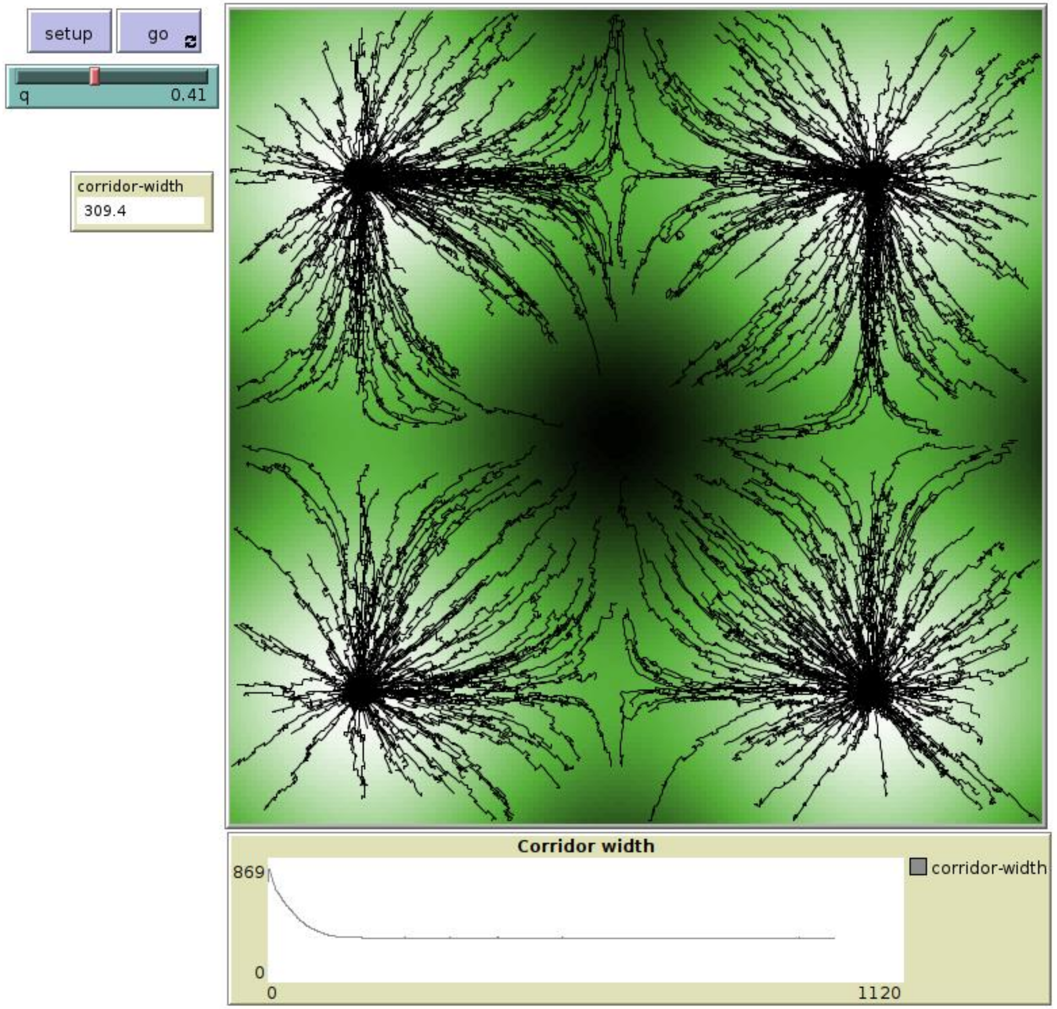
- OED: **emergence.** An unforeseen occurrence.
- OED: **emerge.** Come into being with the passage of events.
- Railsback & Grimm Chapter 8: **emerge.** "Arise in relatively complex and unpredictable ways"
- Cf **impose**: "forced to occur in direct and predictable ways"
- R&G **unpredictable**: "outcomes difficult or impossible to predict just by thinking"
- Explain things by simulation - "can you grow it?"

Qualitative criteria for emergence (Railsback and Grimm)

- It is not simply the sum of the properties of the model's individuals
- It is a different type of result than individual-level properties or decisions
- It cannot easily be predicted from the properties of the individuals

Butterfly corridors – a good example of emergence?

- It is not simply the sum of the properties of the model's individuals
  - Yes-ish – corridors are perhaps a bit more than a sum of butterfly locations
- It is a different type of result than individual-level properties or decisions
  - Yes
- It cannot easily be predicted from the properties of the individuals
  - We would probably expect more noise to give wider corridors – do they?



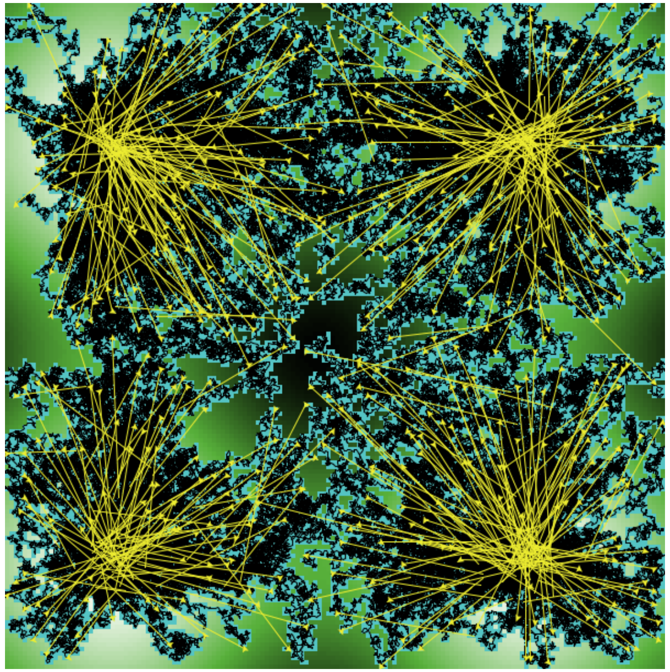
Quantifying corridor width

Corridor width =

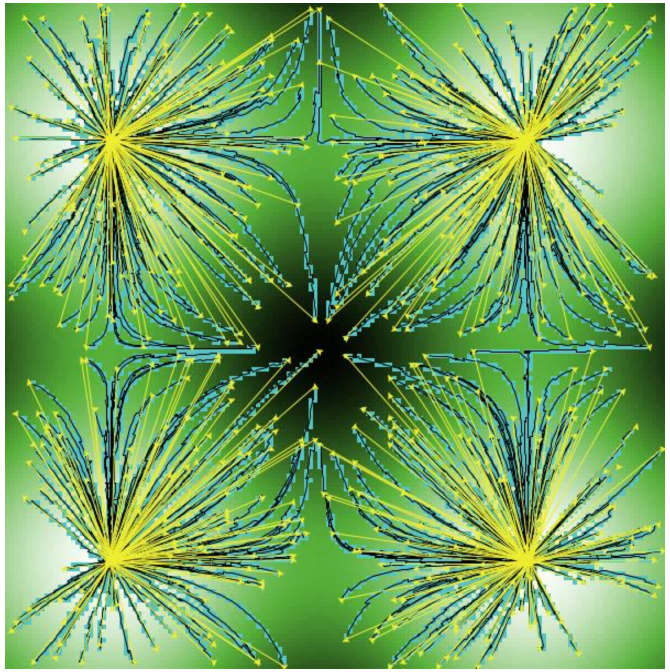
(#patches visited)/

(Mean distance between starting point and end point of each butterfly)

q=0.04 Corridor width = 663

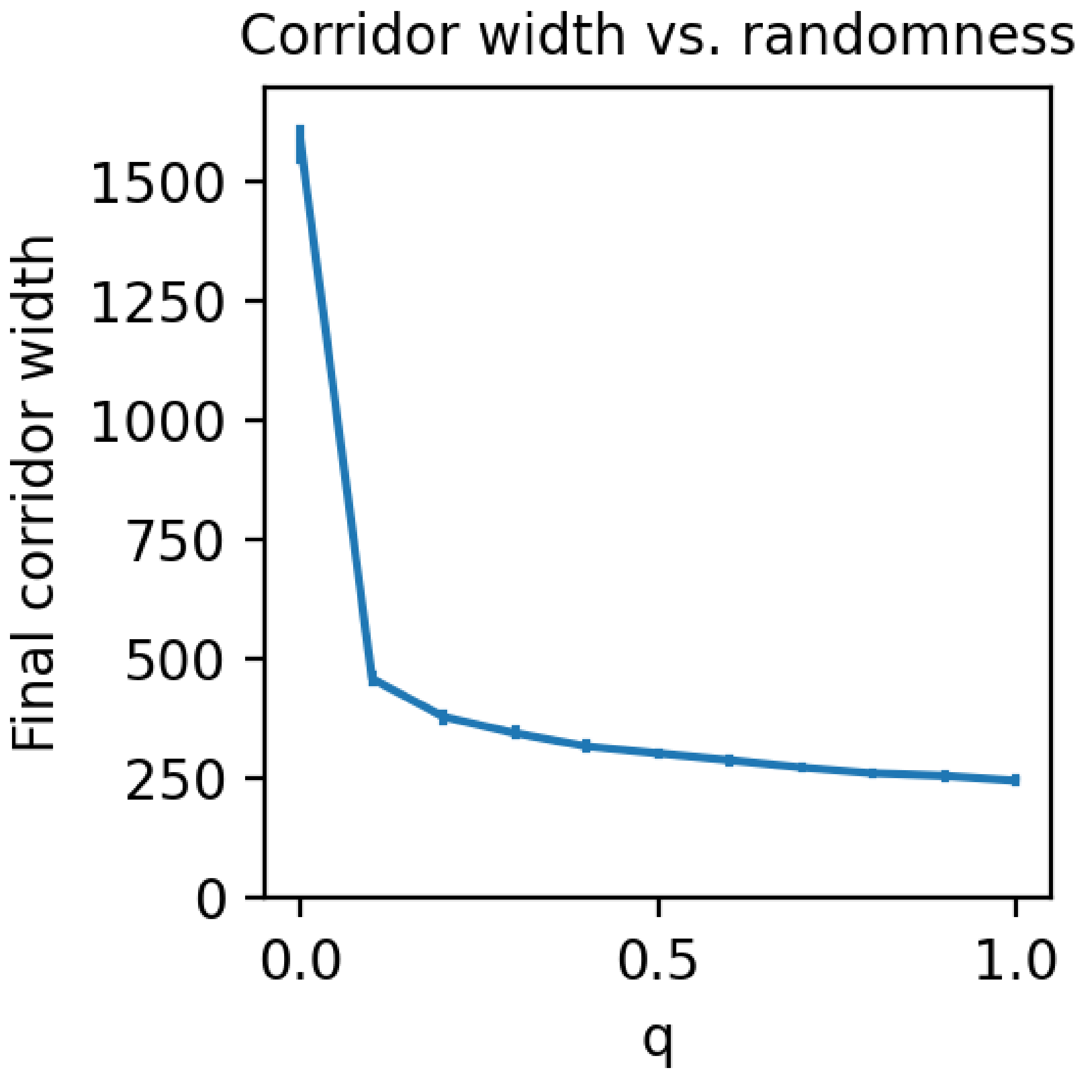


q=0.79 Corridor width = 269



Butterfly corridor width as function of randomness

- Results of running an experiment using NetLOGO BehaviourSpace:
  - Parameter *q* varied in steps of 0.1
  - 10 replications for each value of *q*
  - Data exported and plotted using Python/Matplotlib/Seaborn
  - Mean and S.D. shown
- Corridor width increases with increasing randomness (decreasing *q*)
  - as expected, so perhaps not very emergent – but agent behaviour is very simple.

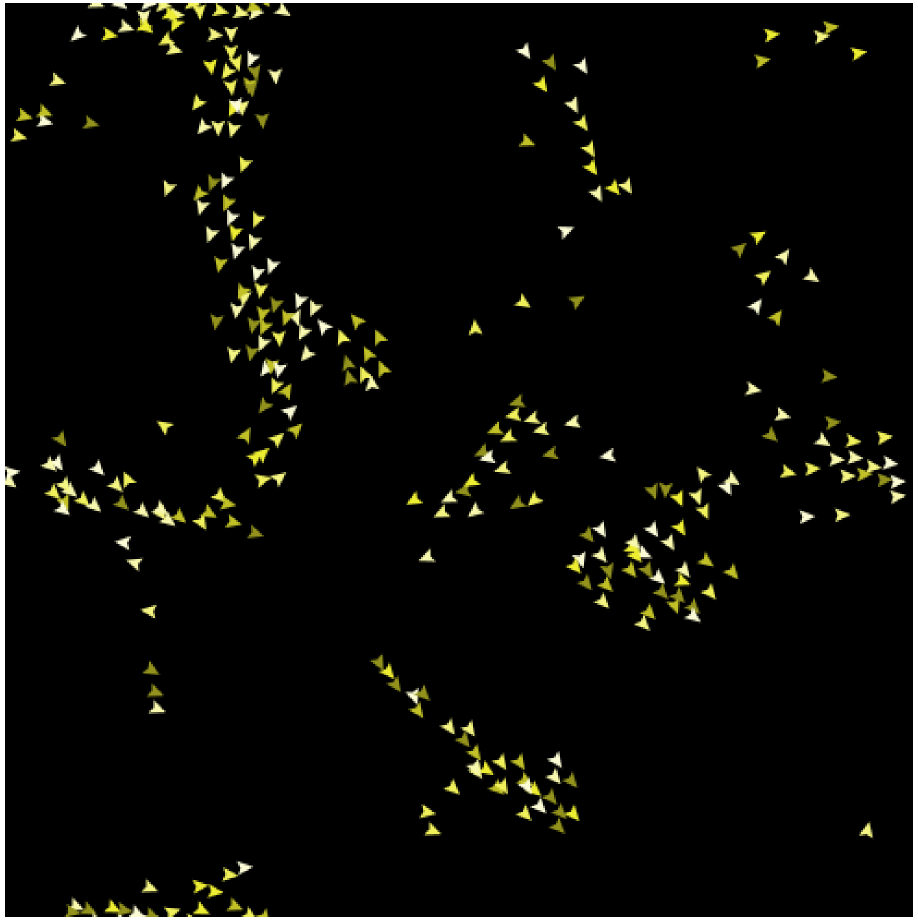


The ideal amount of emergence?

- Highly imposed results, not enough emergence
  - Too predictable
- Very complex model, complex emergent behaviours
  - Too difficult to understand and learn from
- Complex behaviours emerge from apparently simple rules
  - Just right!
- Crucial that the model is appropriate for question

The Three Bears, pictured by John R Neill (1908). Chicago: Reilly & Britton

A good example of emergence? - The NetLOGO flocking model



- It is not simply the sum of the properties of the model's individuals?
- It is a different type of result than individual-level properties or decisions?
- It cannot easily be predicted from the properties of the individuals?

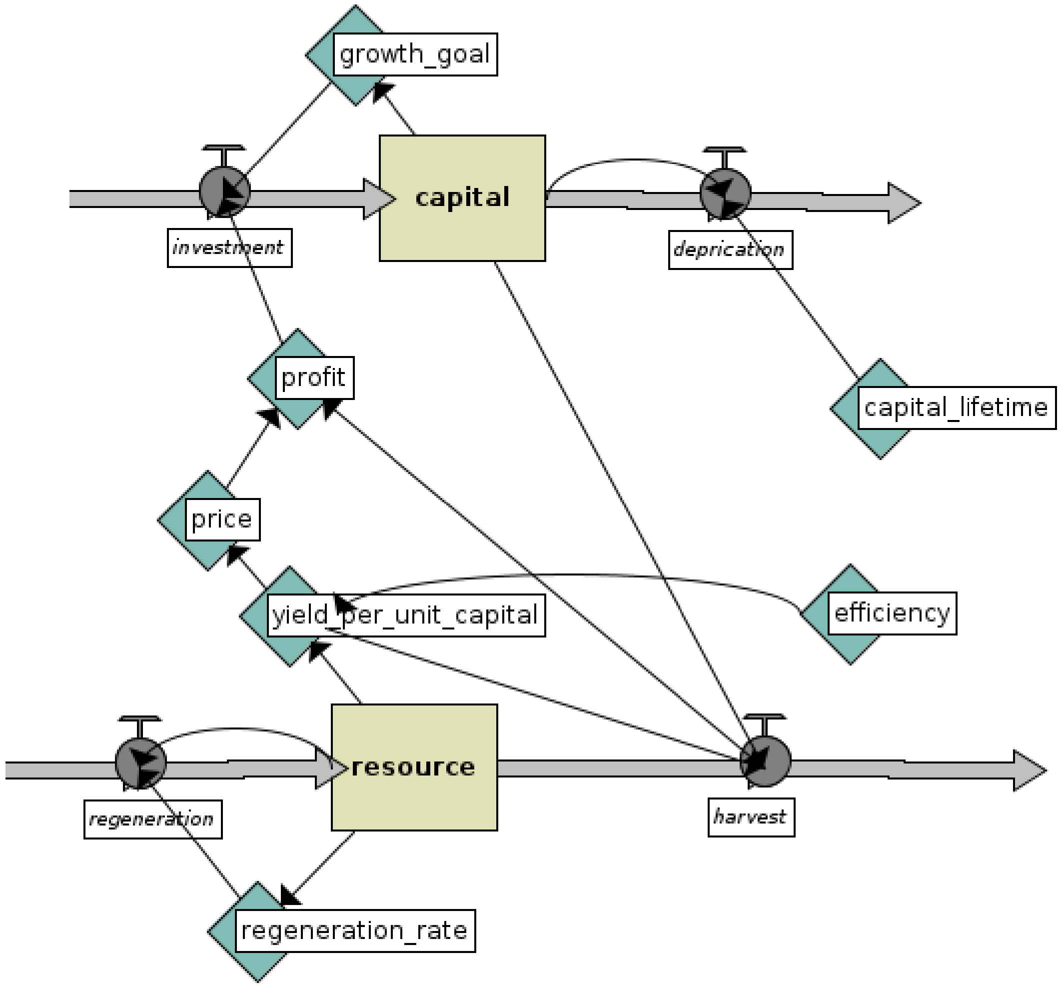
Properties of dynamical systems

Deterministic dynamical system (system dynamics)

- System dynamics models are a special case of a mathematical dynamical system composed of differential equations
- Such systems have characteristic behaviours:
  - Steady state behaviour
  - Oscillating behaviour
  - For 3 stocks (state variables) or more, chaotic behaviour
- The **same model** can exhibit more than one behaviour depending on **its parameter** setting

Example 1: Fishing economy (Meadows Chapter 2, Fig 42)

- Renewable **resource** stock: fish
- Renewable **capital** stock: fishing boats
- Fish **harvest** depends on **capital**, **resource** and **efficiency** (a parameter)
- **Price** gets higher when there is scarcity
- More **profit** => more **investment** => more **capital** => more **harvest** => more **profit**
  - But more **harvest** => less **resource**

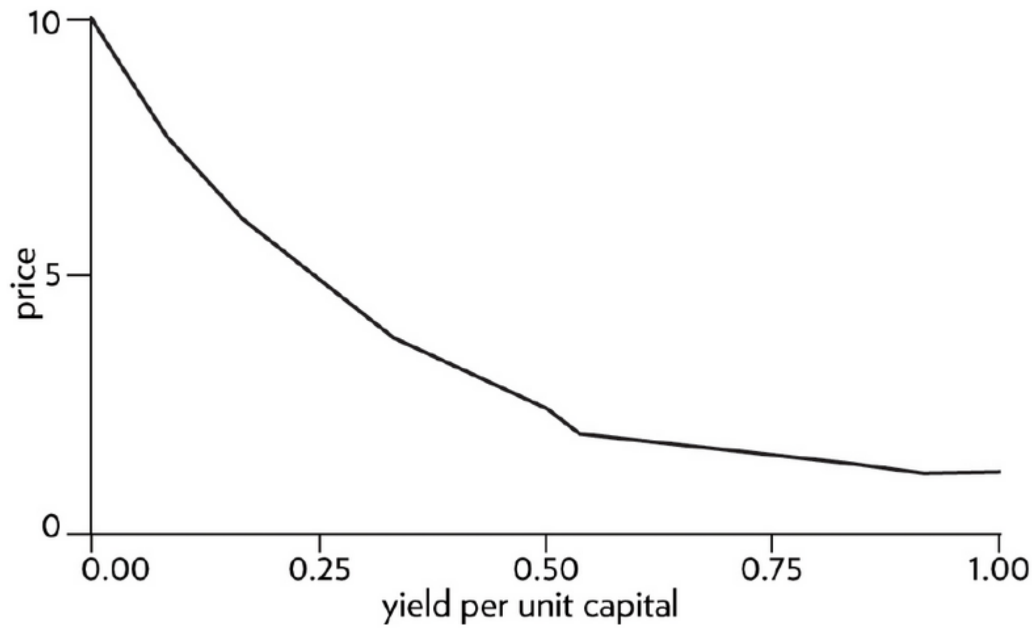


Aside: replication

Meadows, *Thinking in Systems*, p. 227

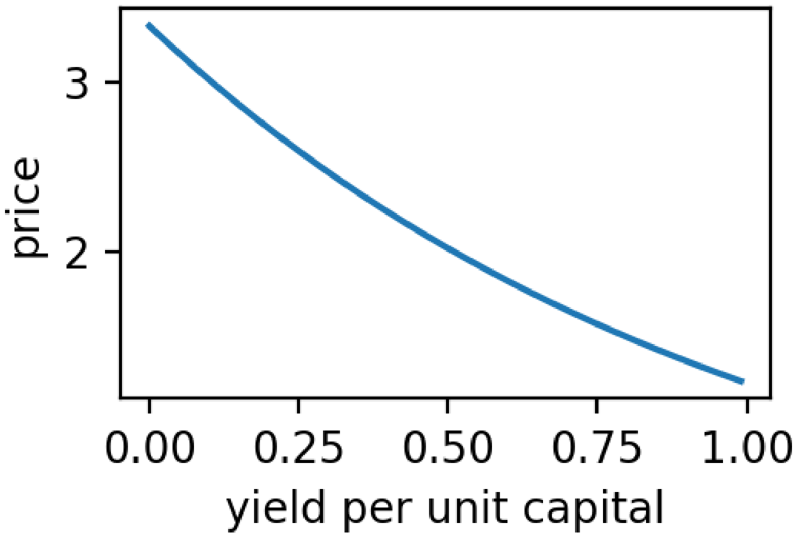
profit = (*price* × *harvest*) – *capital*

price starts at 1.2 when yield per unit capital is high and rises to 10 as yield per unit capital falls. This is the same nonlinear relationship for price and yield as in the previous model.

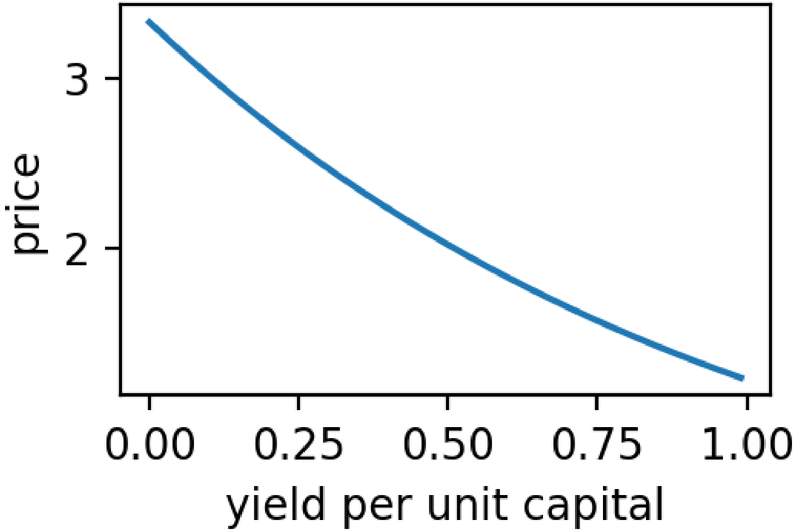


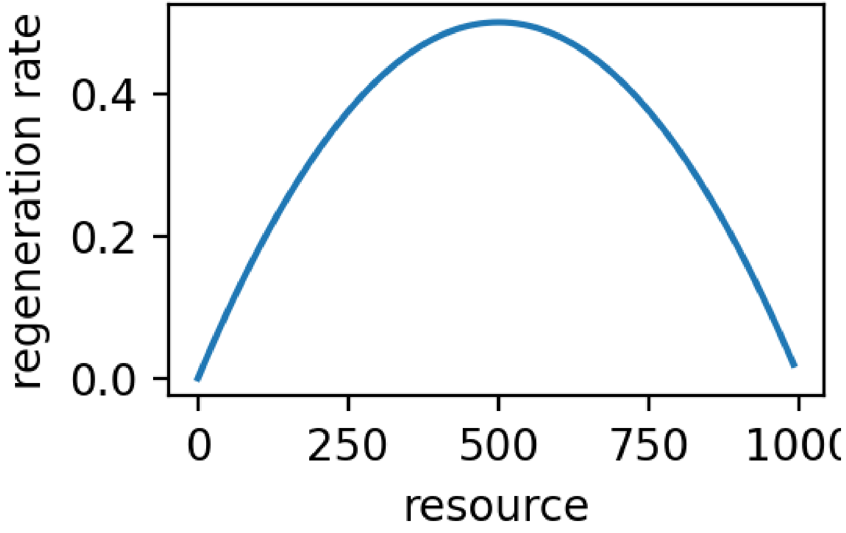
Me

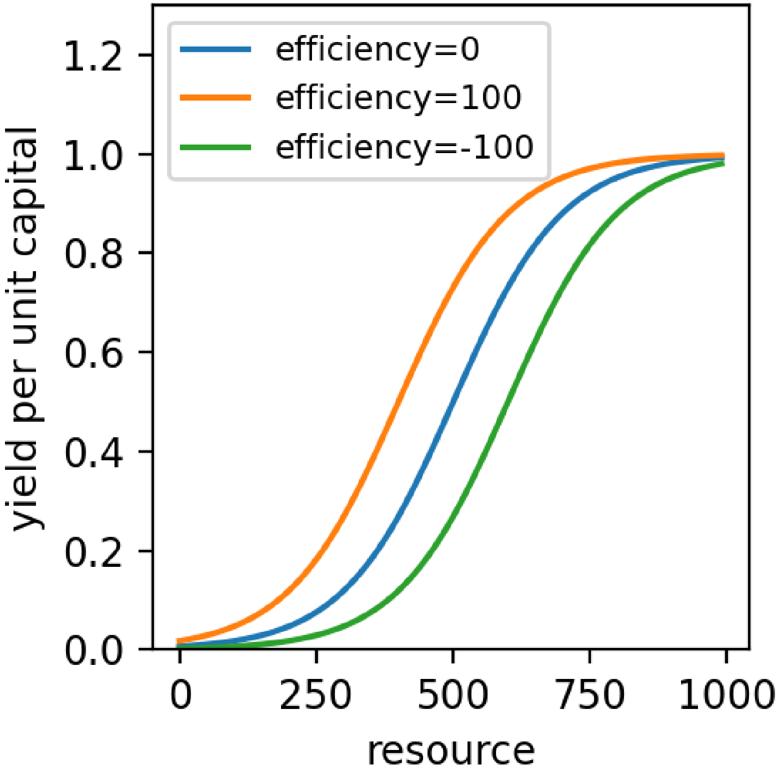
- Created a function that I *thought* was the same, but wasn't!
- Lesson: plot your functions before simulating

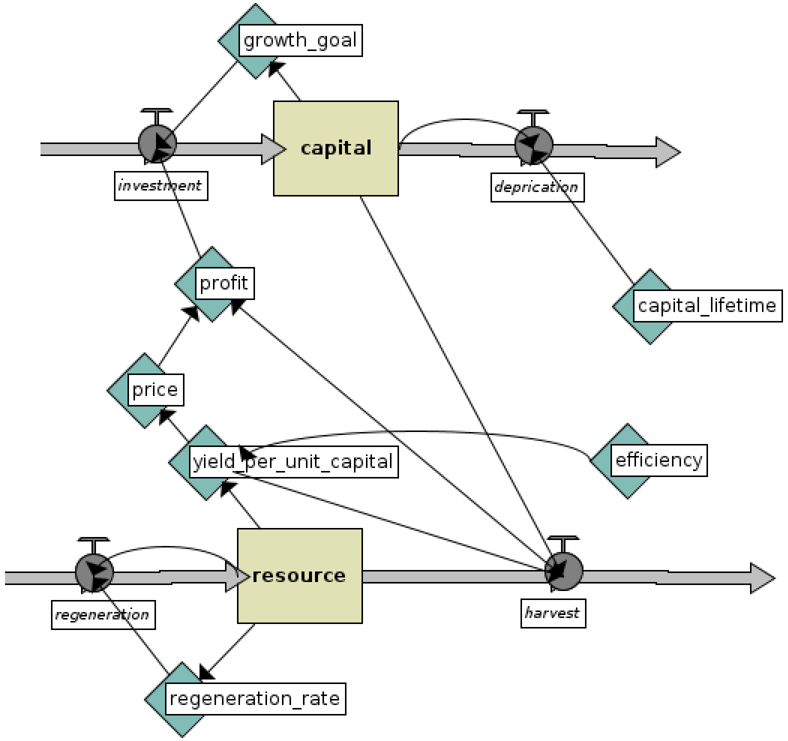


Some key functions





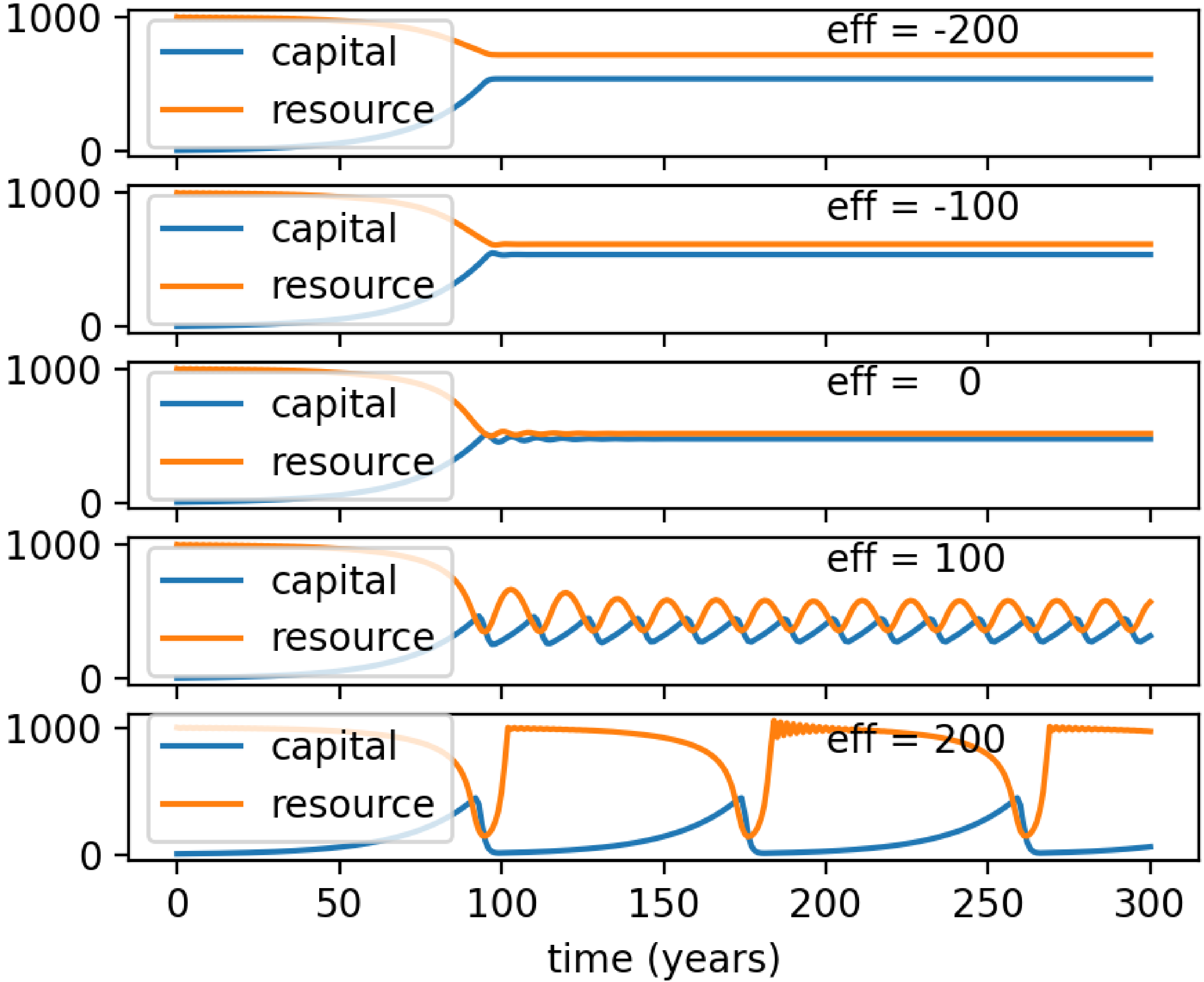




Parameter Search

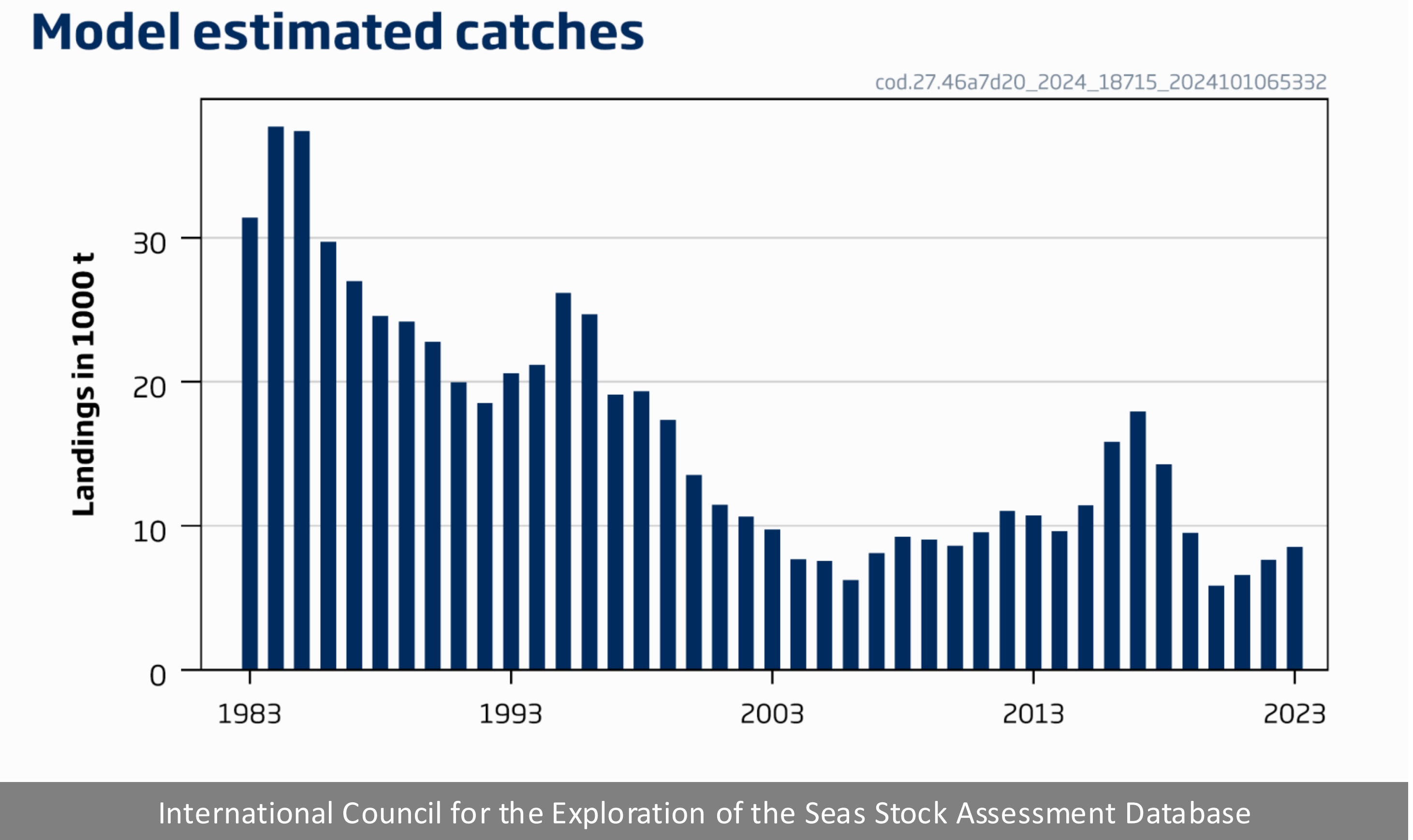
- Increase the efficiency parameter from low to high using BehaviourSpace
- What happens?

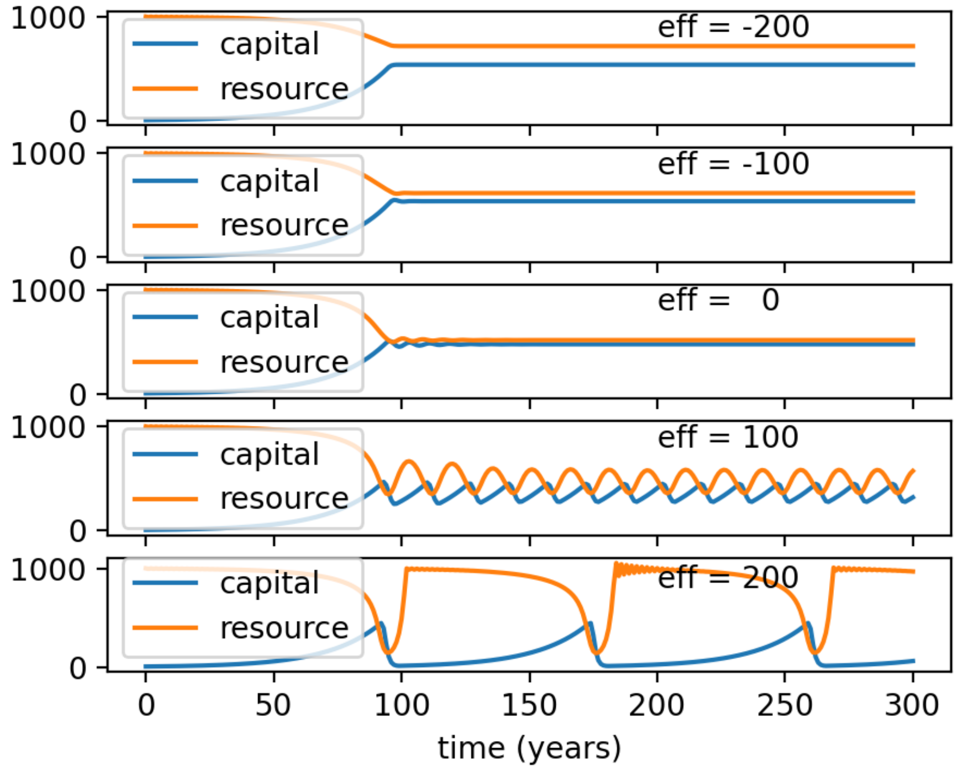
- Note **dynamic stability**



Data versus model

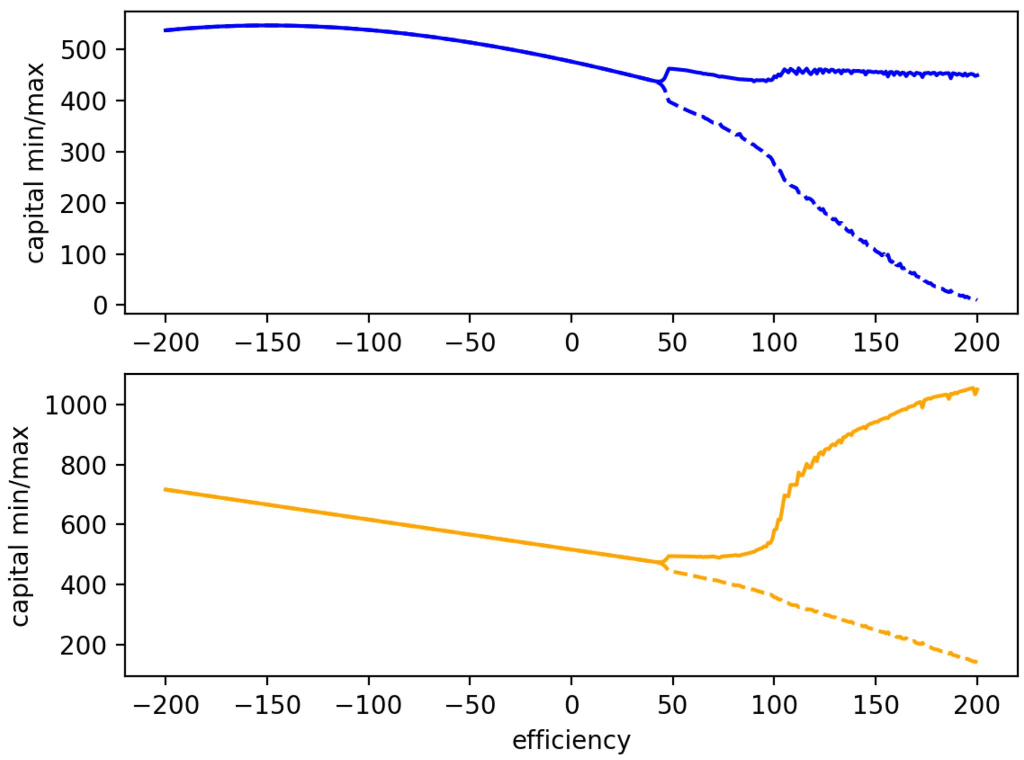
- Cod in Subarea 4, divisions 6.a and 7.d, and Subdivision 20 (North Sea, West of Scotland, eastern English Channel and Skagerrak)





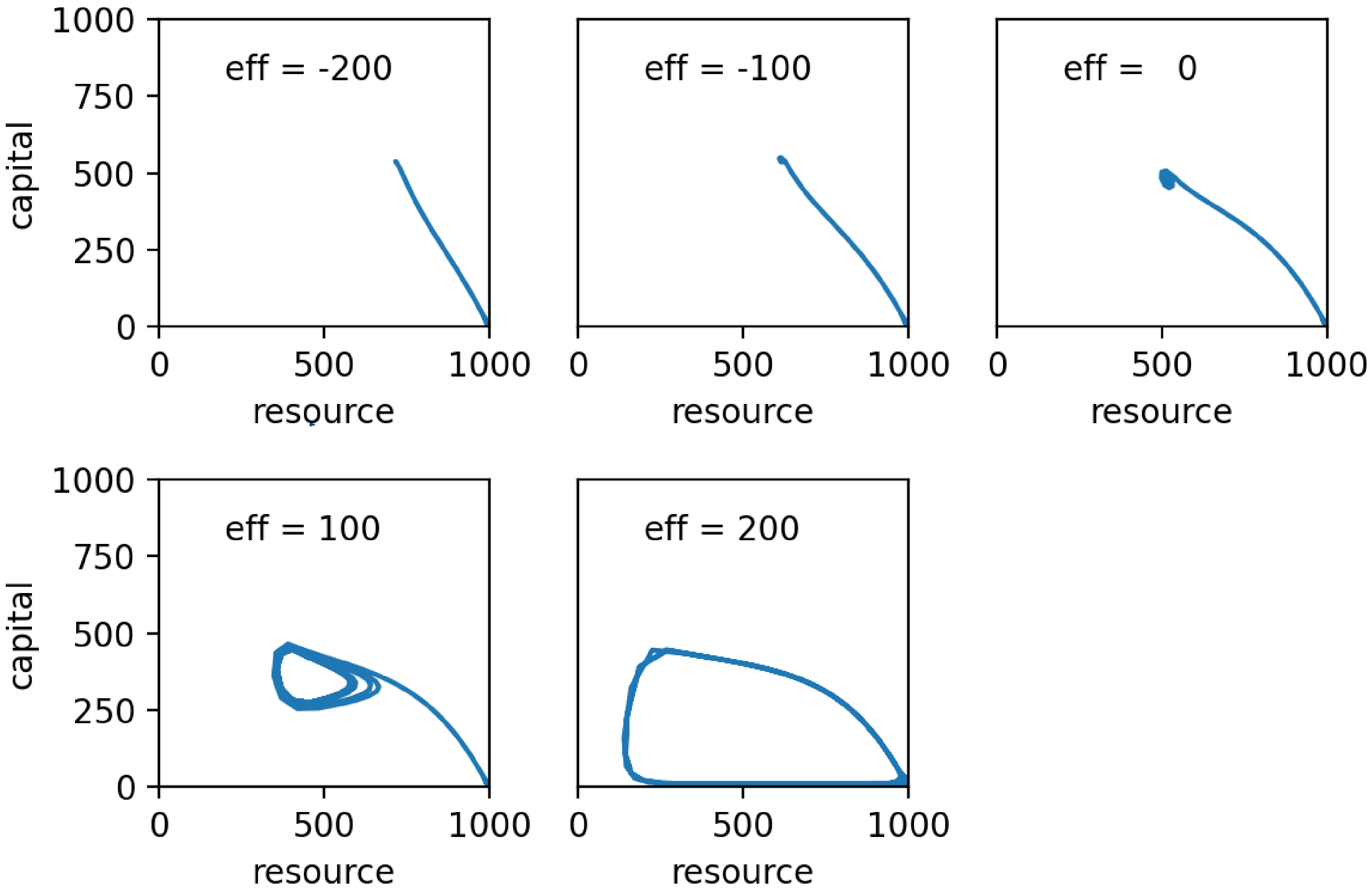
Tipping point

- There is a **control parameter** that is gradually adjusted or changes
  - E.g. increasing efficiency
- At some value of the parameter the behaviour suddenly changes
  - E.g. an equilibrium point changes to a new value
  - Or the dynamical behaviour of the system changes
- In **System Dynamics** models this is a "**Tipping point**"
- Known as a "**Bifurcation**" in the mathematical field of **dynamical systems**

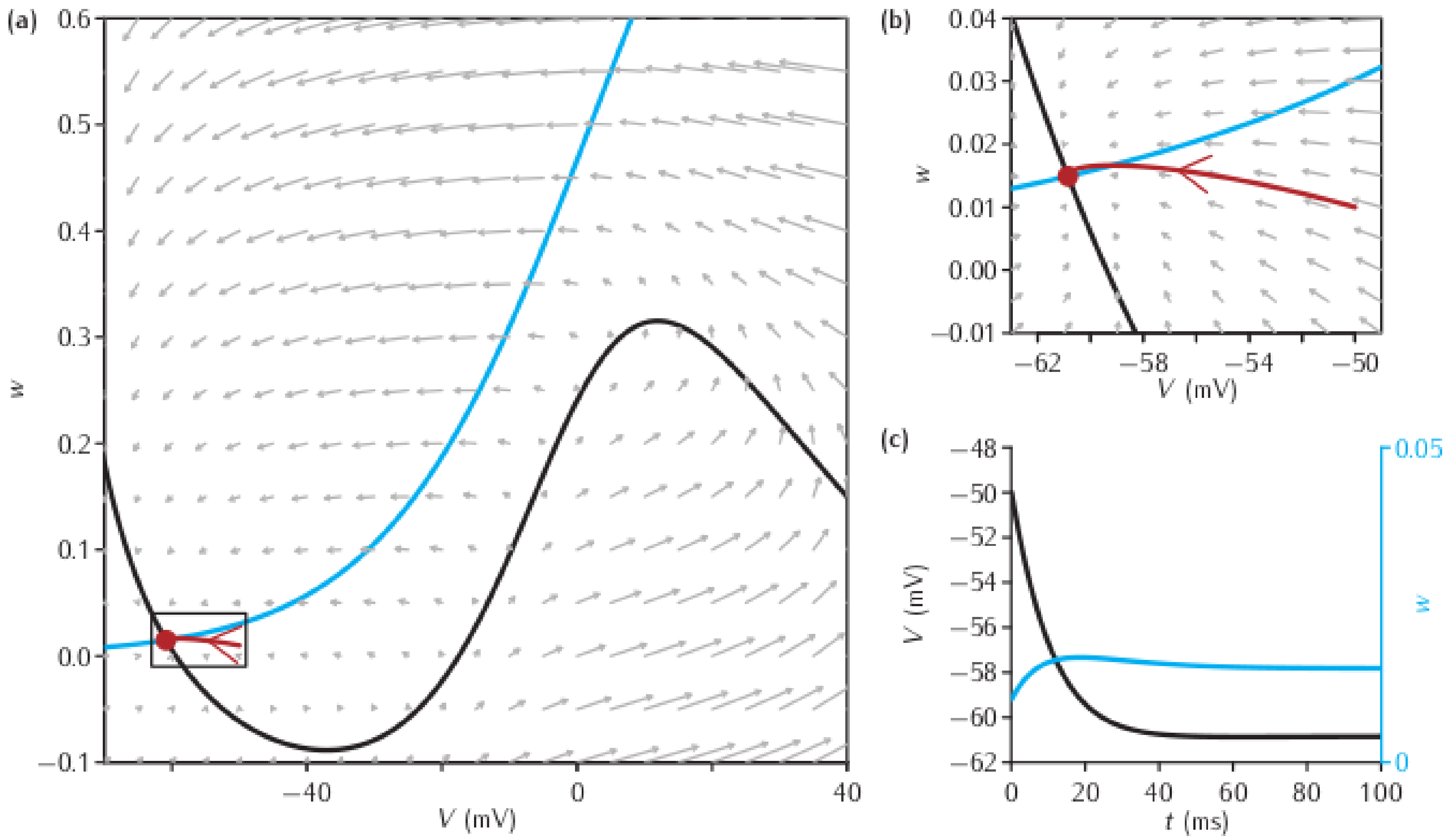


Phase planes

- Equilibrium points, or **fixed points**, corresponding to **steady state** solutions
- These equilibrium points can be **stable** or **unstable**
- **Limit cycles,** i.e. oscillations

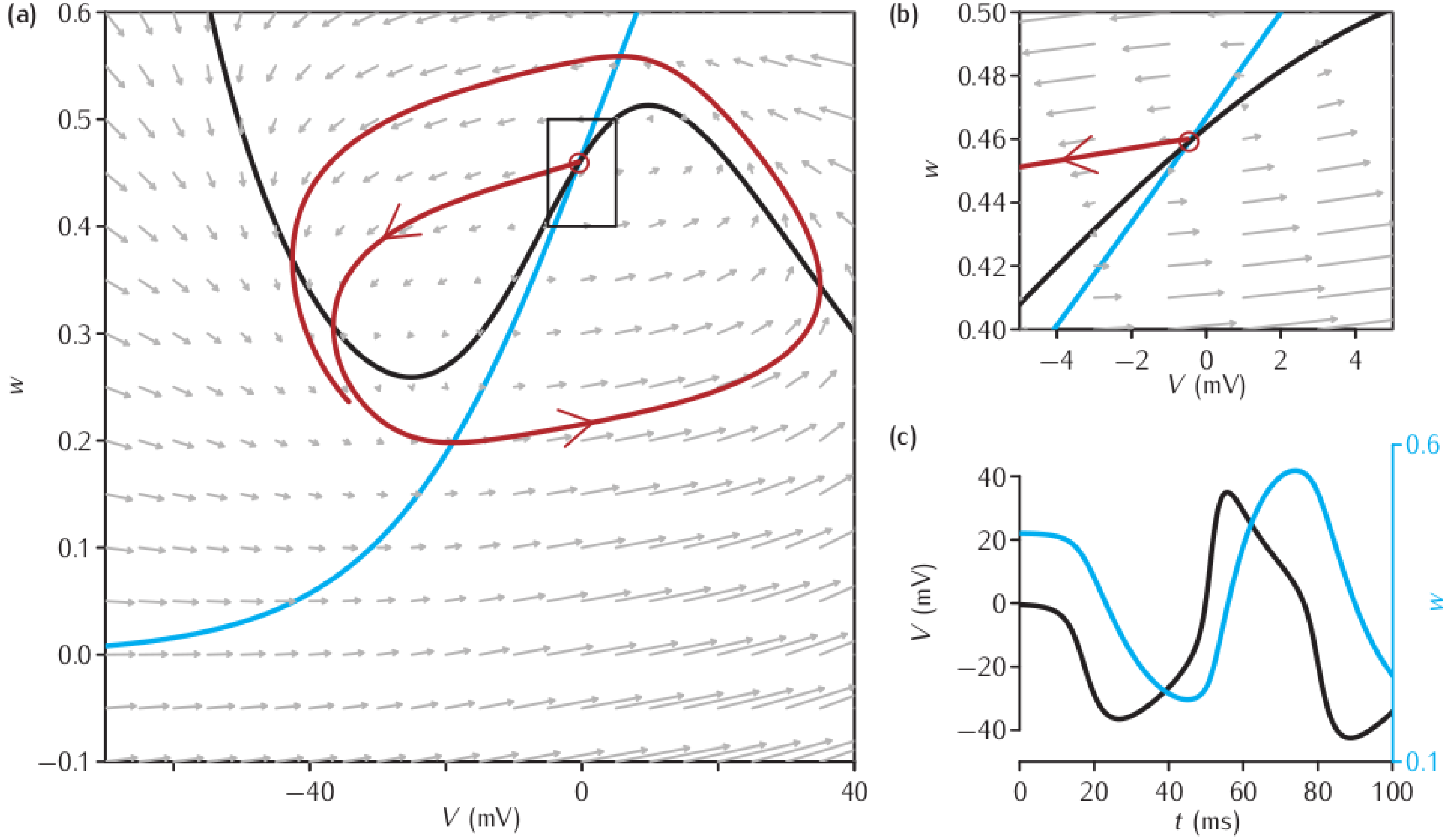


Stable fixed point



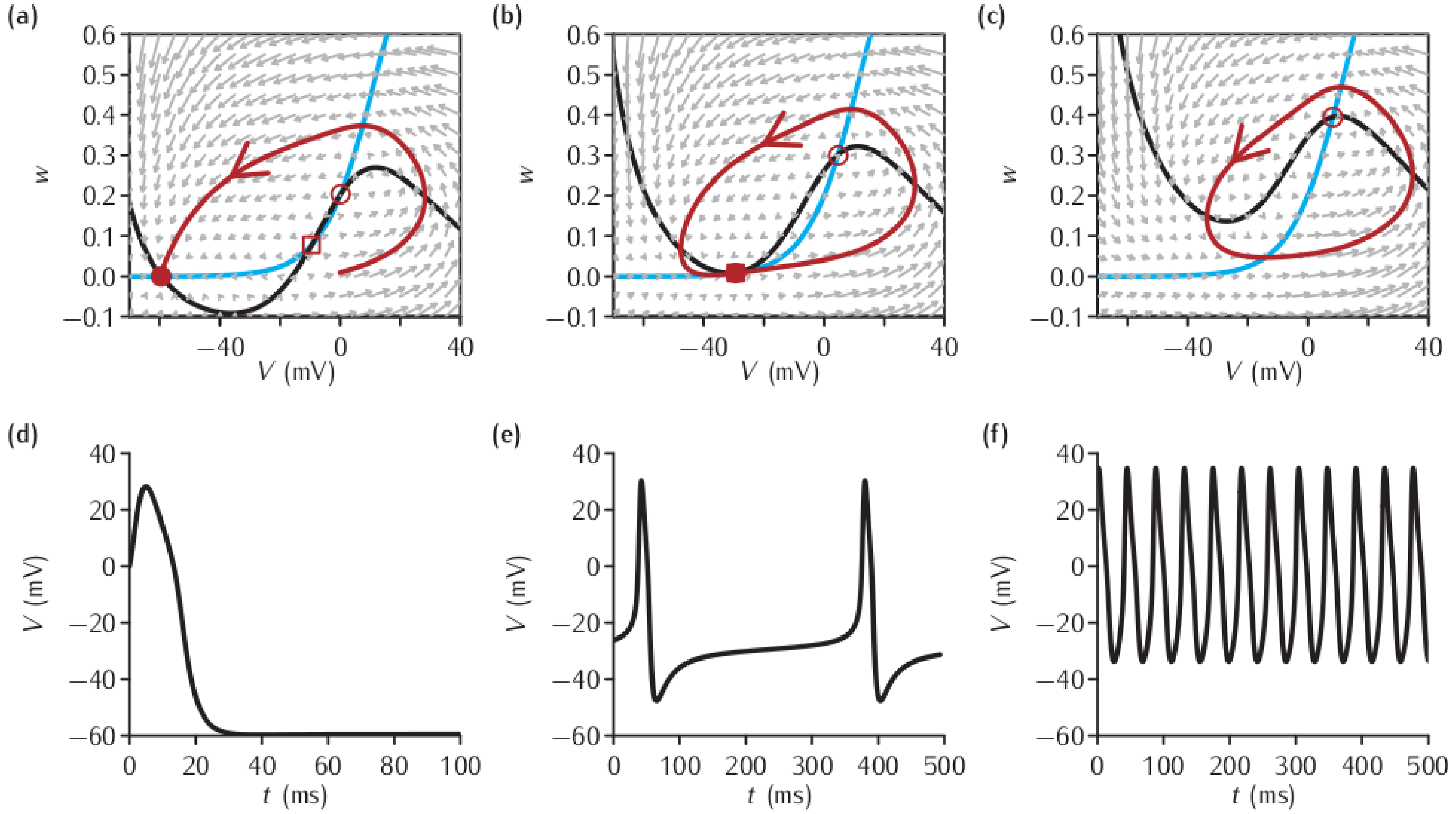
Sterratt, Graham, Gillies, Einevoll & Willshaw (2023), *Principles of Computational Modelling in Neuroscience*, CUP.

Starting from an unstable fixed point and going into a limit cycle

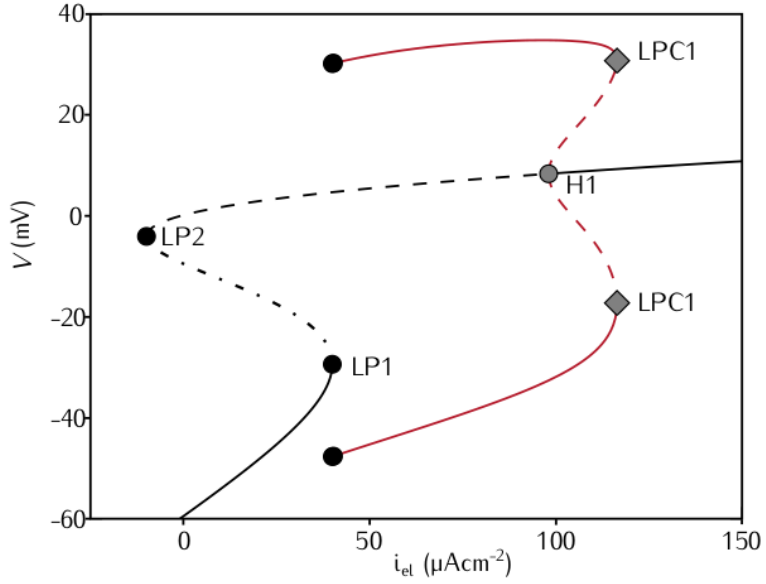


Sterratt, Graham, Gillies, Einevoll & Willshaw (2023), *Principles of Computational Modelling in Neuroscience*, CUP.

A saddle-node bifurcation



Bifurcation diagram

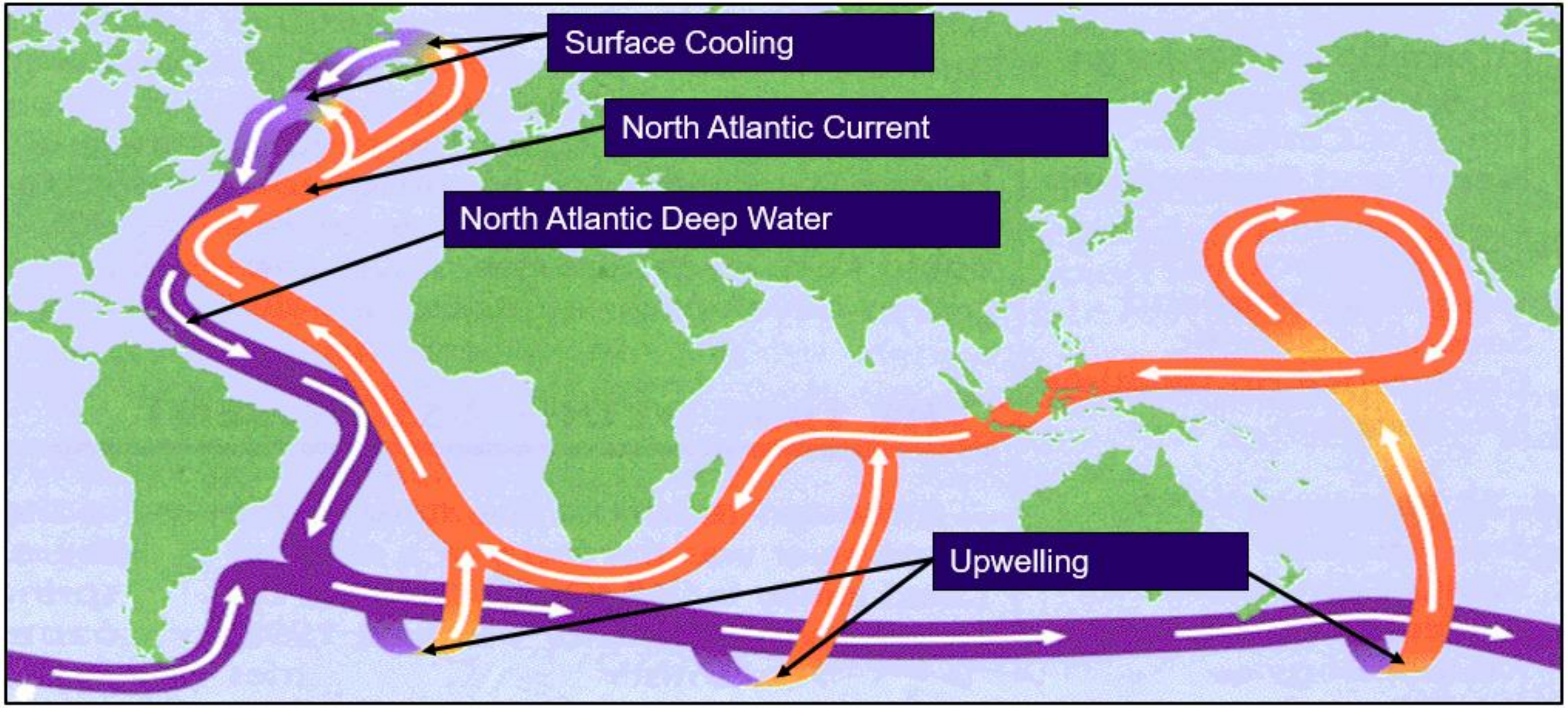


Sterratt, Graham, Gillies, Einevoll & Willshaw (2023), *Principles of Computational Modelling in Neuroscience*, CUP.

Hysteresis

- Just taking the control parameter back past the tipping point value doesn't put the system back into the old state
  - E.g. lowering current (in the previous case) might not immediately restore the old behaviour
- This behaviour is called **hysteresis loop**

Example 2: Atlantic Meridional Overturning Circulation

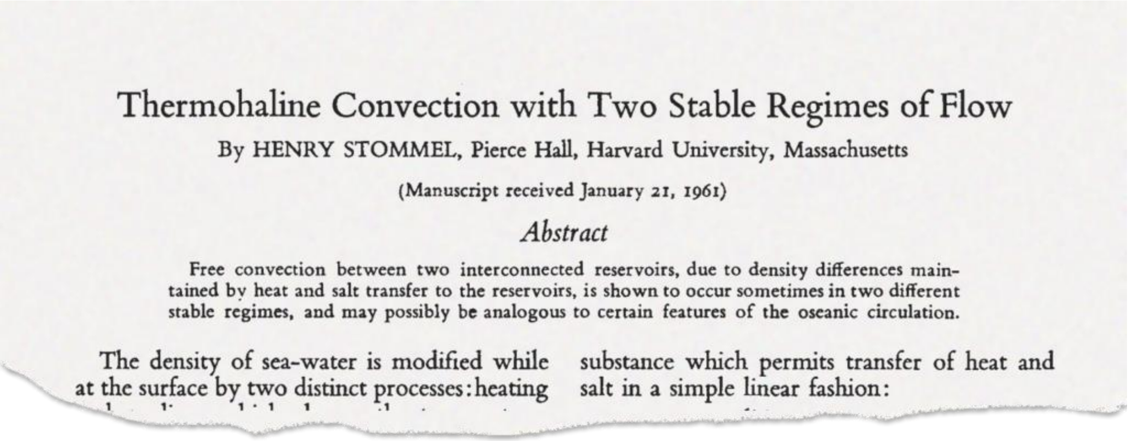


Met Office Hadley Centre Climate Briefing Note

https://www.metoffice.gov.uk/binaries/content/assets/metofficegovuk/pdf/weather/learn-about/climate/ocean-and-cryosphere-report/srocc_amoc.pdf

Example 2: Atlantic Meridional Overturning Circulation

- Stommel (1961) two-box model



Thermohaline Convection with Two Stable Regimes of Flow

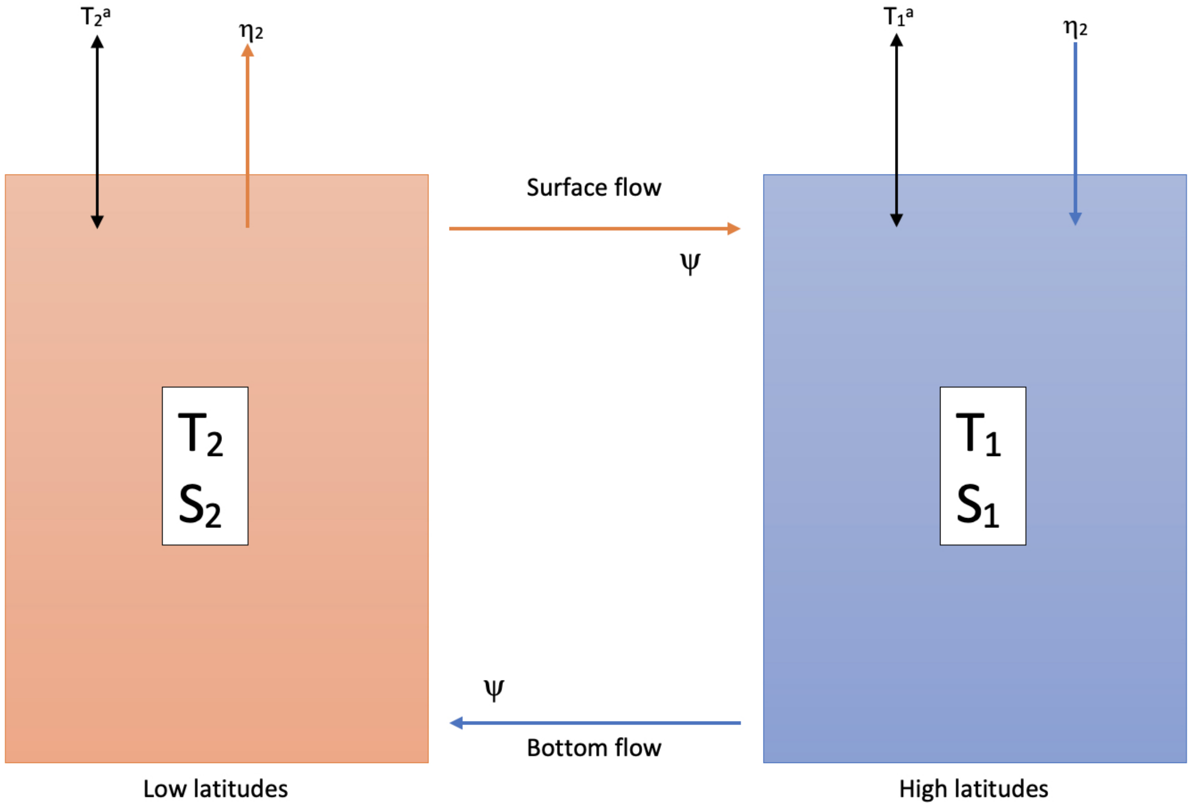
By HENRY STOMMEL, Pierce Hall, Harvard University, Massachusetts

(Manuscript received January 21, 1961)

Abstract

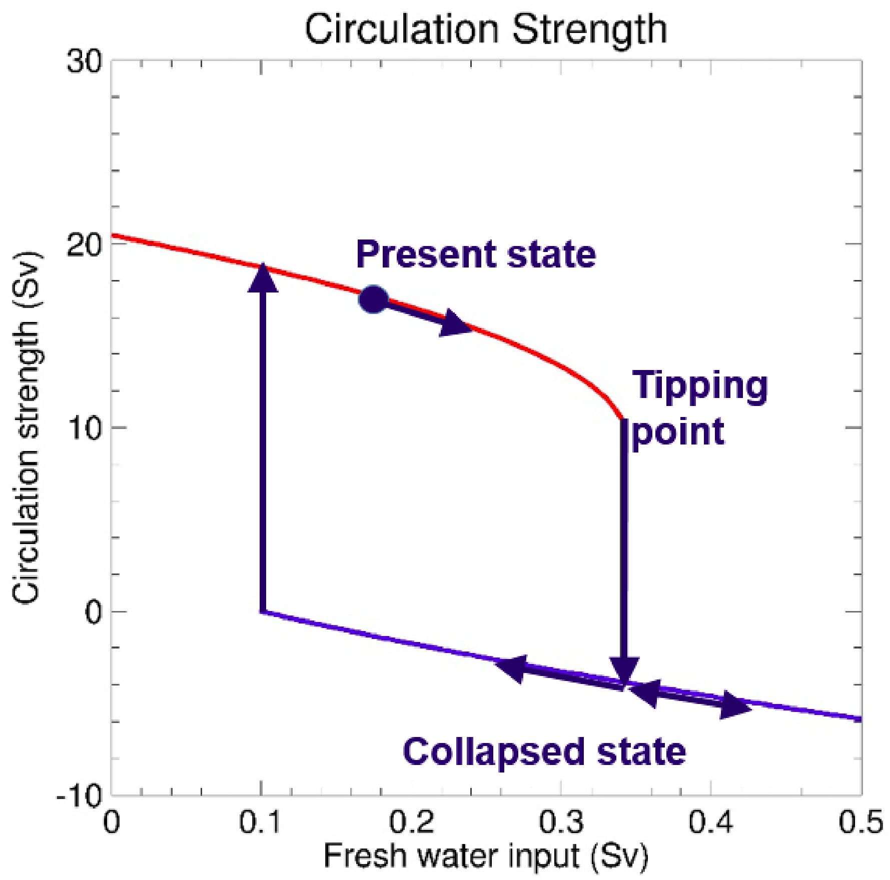
Free convection between two interconnected reservoirs, due to density differences maintained by heat and salt transfer to the reservoirs, is shown to occur sometimes in two different stable regimes, and may possibly be analogous to certain features of the oceanic circulation.

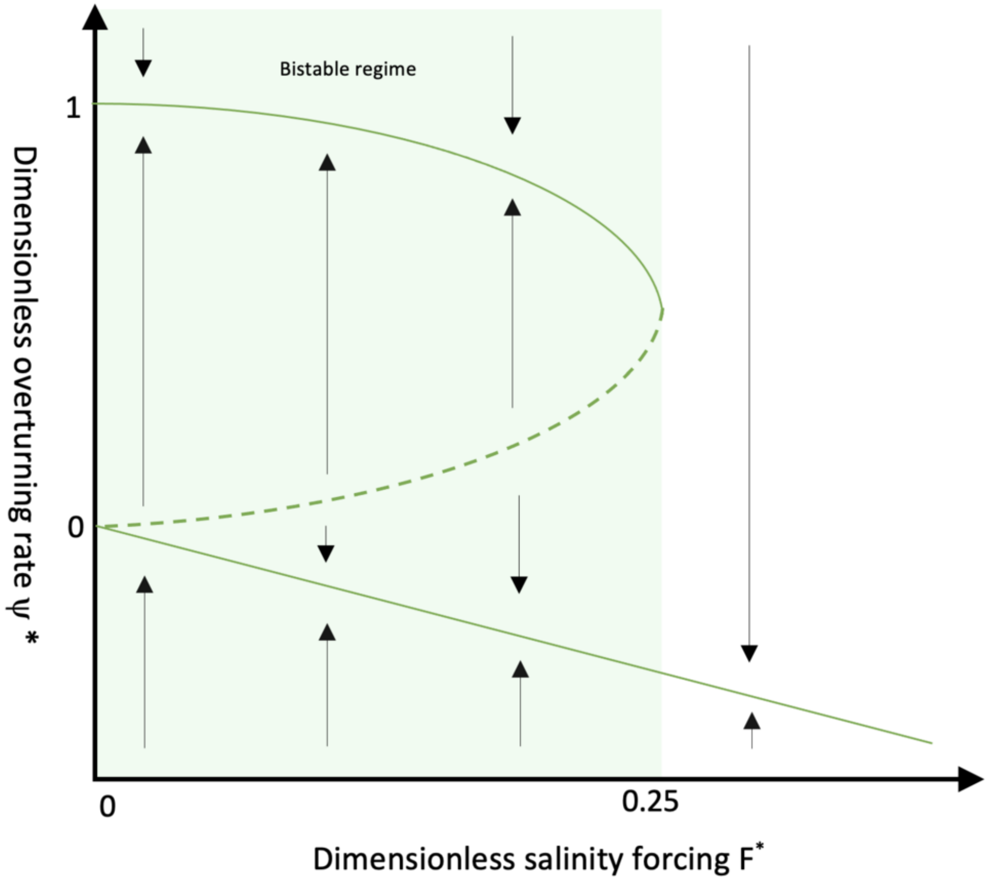
The density of sea-water is modified while at the surface by two distinct processes: heating substance which permits transfer of heat and salt in a simple linear fashion:



[Wikipedia article on Multiple equilibria in the Atlantic meridional overturning circulation]

Bifurcation diagrams





Met office
https://www.metoffice.gov.uk/binaries/content/assets/metofficegovuk/pdf/weather/learn-about/climate/deliverables/thresholds-and-feedbacks-introduction.pdf

[Wikipedia article on Multiple equilibria in the Atlantic meridional overturning circulation]

Summary

- **D**esign concept: Emergence
- Model analysis: Properties of dynamical systems
  - Effect of changing parameter
  - Dynamical states
  - Bifurcations
  - Phase plots
  - Application to ecological-economic model and earth model
- Some quantification of patterns and exploration of parameter space
- Next:
  - Tutorial (Thursday): Paper mini-presentations
  - Lab (Thursday): Butterfly hilltopping model