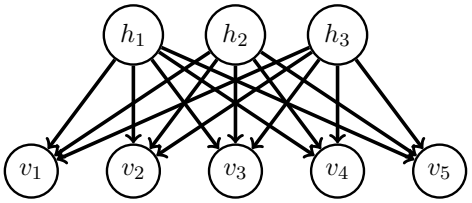
Probabilistic Machine Learning Notes (FA and ICA)

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These notes summarise selected lecture concepts and are not a substitute for working through the lecture slides, tutorials, and self-study exercises. Feel free to personalise and develop them into your own summary sheet.

Factor analysis — A graphical model where statistical dependencies between the observed variables (visibles $\mathbf{v}$) is modelled through unobserved variables (latents $\mathbf{h}$). In factor analysis, the latents $\mathbf{h}$ are assumed to be independent Gaussians with zero mean and unit variance.

$$
\begin{aligned}
p(\mathbf{h}) &= \mathcal{N}(\mathbf{h}; \mathbf{0}, \mathbf{I}) \\
p(\mathbf{v} \mid \mathbf{h}; \boldsymbol{\theta}) &= \mathcal{N}(\mathbf{v}; \mathbf{F}\mathbf{h} + \mathbf{c}, \boldsymbol{\Psi}) \\
\\
\mathbf{v} &= \mathbf{F}\mathbf{h} + \mathbf{c} + \boldsymbol{\epsilon} \\
\boldsymbol{\epsilon} &\sim \mathcal{N}(\boldsymbol{\epsilon}; 0, \boldsymbol{\Psi})
\end{aligned}
$$



The covariance matrix $\boldsymbol{\Psi}$ is a diagonal matrix. Probabilistic PCA is a special case of factor analysis, where $\boldsymbol{\Psi} = \sigma^2\mathbf{I}$.

Independent component analysis — The DAG is the same as in factor analysis, but with non-Gaussian latents (one latent may be Gaussian)

$$
\begin{aligned}
p(\mathbf{h}) &= \prod_i p(h_i) \\
p(\mathbf{v} \mid \mathbf{h}; \boldsymbol{\theta}) &= \mathcal{N}(\mathbf{v}; \mathbf{A}\mathbf{h} + \mathbf{c}, \boldsymbol{\Psi})
\end{aligned}
$$