

These notes summarise selected lecture concepts and are not a substitute for working through the lecture slides, tutorials, and self-study exercises. Feel free to personalise and develop them into your own summary sheet.

Statistical independence — $\mathbf{x}$ and $\mathbf{y}$ are statistically independent iff for all values of $\mathbf{x}$ and $\mathbf{y}$

$$p(\mathbf{x}, \mathbf{y}) = p(\mathbf{x})p(\mathbf{y}) \quad \text{or} \quad (1)$$

$$p(\mathbf{x}|\mathbf{y}) = p(\mathbf{x}) \quad (\text{whenever } p(\mathbf{y}) > 0) \quad (2)$$

Notation: $\mathbf{x} \perp\!\!\!\perp \mathbf{y}$.

Conditional statistical independence — $\mathbf{x}$ and $\mathbf{y}$ are conditionally independent given $\mathbf{z}$ iff for all possible values of $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$:

$$p(\mathbf{x}, \mathbf{y}|\mathbf{z}) = p(\mathbf{x}|\mathbf{z})p(\mathbf{y}|\mathbf{z}) \quad (\text{whenever } p(\mathbf{z}) > 0) \quad \text{or} \quad (3)$$

$$p(\mathbf{x}|\mathbf{y}, \mathbf{z}) = p(\mathbf{x}|\mathbf{z}) \quad (\text{whenever } p(\mathbf{y}, \mathbf{z}) > 0) \quad (4)$$

Notation: $\mathbf{x} \perp\!\!\!\perp \mathbf{y}|\mathbf{z}$.

Statistical independence of multiple random variables — Variables $\mathbf{x}_1, \dots, \mathbf{x}_n$ are (conditionally) independent iff for every partition of the index set $\{1, \dots, n\}$ into disjoint subsets A and B , the random vectors $\mathbf{x}_A$ and $\mathbf{x}_B$ are (conditionally) independent.

Autoregressive model — In the chain rule $p(\mathbf{x}) = \prod_{i=1}^d p(x_i|\text{pre}_i)$, each $p(x_i|\text{pre}_i)$ is specified as a member of a parametric family.