

These are exercises for self-study and exam preparation. All material is examinable unless otherwise mentioned.

Exercise 1. *The chain rule factorisation*

Let (x, y, z) be three discrete random variables, each taking values in $\{0, 1\}$. Suppose their joint probability mass function is

x	y	z	$p(x, y, z)$
0	0	0	$\frac{1}{20}$
0	0	1	$\frac{2}{20}$
0	1	0	$\frac{3}{20}$
0	1	1	$\frac{4}{20}$
1	0	0	$\frac{1}{20}$
1	0	1	$\frac{3}{20}$
1	1	0	$\frac{2}{20}$
1	1	1	$\frac{4}{20}$

- (a) The chain rule for the ordering x, y, z is

$$p(x, y, z) = p(x)p(y|x)p(z|x, y) \quad (1)$$

Determine the numerical values for all (conditional) pmfs in the above factorisation.

- (b) Specify the chain-rule factorisation for the ordering z, y, x .
- (c) Compute the numerical values for each factor found in the factorisation above.
- (d) Check that the pmf for $(x, y, z) = (1, 1, 0)$ is the same when computed with the chain-rule for the ordering x, y, z or z, y, x .

Exercise 2. *Expectations*

Let (x, y, z) be three discrete random variables, each taking values in $\{0, 1\}$. Suppose their joint probability mass function is

x	y	z	$p(x, y, z)$
0	0	0	$\frac{1}{20}$
0	0	1	$\frac{2}{20}$
0	1	0	$\frac{3}{20}$
0	1	1	$\frac{4}{20}$
1	0	0	$\frac{1}{20}$
1	0	1	$\frac{3}{20}$
1	1	0	$\frac{2}{20}$
1	1	1	$\frac{4}{20}$

- (a) Compute $\mathbb{E}[(x + 1)^2]$
- (b) Compute $\mathbb{E}[(x + 1)^2(y + 2)]$
- (c) Compute $\mathbb{E}[(x + 1)^2(y + 2)|x = 0]$