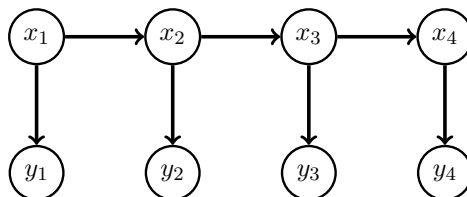


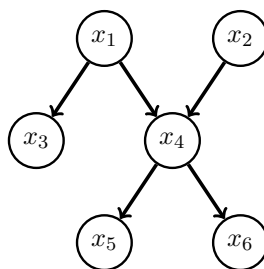
These are exercises for self-study and exam preparation. All material is examinable unless otherwise mentioned.

Exercise 1. Conversion to factor graphs

- (a) Draw an undirected factor graph for the directed graphical model defined by the graph below.

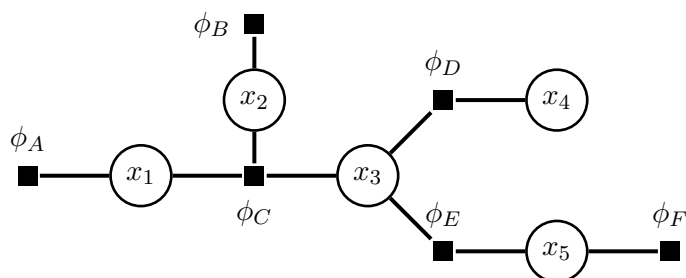


- (b) Draw an undirected factor graph for directed graphical models defined by the graph below (this kind of graph is called a polytree: there are no loops but a node may have more than one parent).



Exercise 2. Sum-product message passing

We here re-consider the factor tree from the lecture on exact inference.



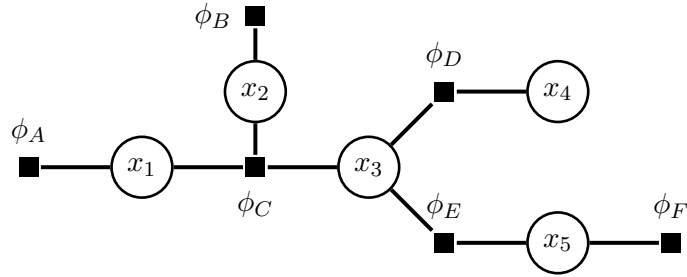
Let all variables be binary, $x_i \in \{0, 1\}$, and the factors be defined as follows:

		x_1	x_2	x_3	ϕ_C	x_3	x_4	ϕ_D	x_3	x_5	ϕ_E	x_5	ϕ_F
		0	0	0	4								
		1	0	0	2								
x_1	ϕ_A	0	1	0	2	0	0	8	0	0	3	0	1
0	2	1	1	0	6	1	0	2	1	0	6	1	8
1	4	0	0	1	2	0	1	2	0	1	6		
		1	0	1	6	1	1	6	1	1	3		
		0	1	1	6								
		1	1	1	4								

- (a) Mark the graph with arrows indicating all messages that need to be computed for the computation of $p(x_1)$.
- (b) Compute the messages that you have identified.
Assuming that the computation of the messages is scheduled according to a common clock, group the messages together so that all messages in the same group can be computed in parallel during a clock cycle.
- (c) What is $p(x_1 = 1)$?
- (d) Draw the factor graph corresponding to $p(x_1, x_3, x_4, x_5 | x_2 = 1)$ and provide the numerical values for all factors.
- (e) Compute $p(x_1 = 1 | x_2 = 1)$, re-using messages that you have already computed for the evaluation of $p(x_1 = 1)$.

Exercise 3. *Max-sum message passing*

We here compute most probable states for the factor graph and factors below.



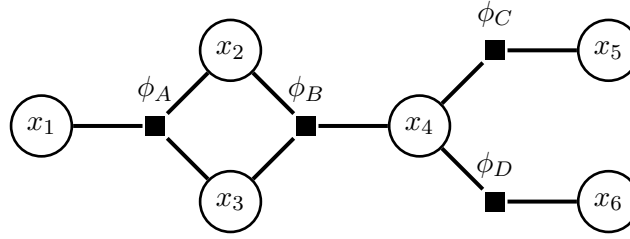
Let all variables be binary, $x_i \in \{0, 1\}$, and the factors be defined as follows:

		x_1	x_2	x_3	ϕ_C	x_3	x_4	ϕ_D	x_3	x_5	ϕ_E	x_5	ϕ_F
		0	0	0	4								
		1	0	0	2								
x_1	ϕ_A	0	1	0	2	0	0	8	0	0	3	0	1
0	2	1	1	0	6	1	0	2	1	0	6	1	8
1	4	0	0	1	2	0	1	2	0	1	6		
		1	0	1	6	1	1	6	1	1	3		
		0	1	1	6								
		1	1	1	4								

- (a) Will we need to compute the normalising constant Z to determine $\text{argmax}_{\mathbf{x}} p(x_1, \dots, x_5)$?
- (b) Compute $\text{argmax}_{x_1, x_2, x_3} p(x_1, x_2, x_3 | x_4 = 0, x_5 = 0)$ via max-sum message passing.
- (c) Compute $\text{argmax}_{x_1, \dots, x_5} p(x_1, \dots, x_5)$ via max-sum message passing with x_1 as sink.
- (d) Compute $\text{argmax}_{x_1, \dots, x_5} p(x_1, \dots, x_5)$ via max-sum message passing with x_3 as sink.

Exercise 4. Choice of elimination order in factor graphs

Consider the following factor graph, which contains a loop:



Let all variables be binary, $x_i \in \{0, 1\}$, and the factors be defined as follows:

x_1	x_2	x_3	ϕ_A	x_2	x_3	x_4	ϕ_B	x_4	x_5	ϕ_C	x_4	x_6	ϕ_D
0	0	0	4	0	0	0	2	0	0	8	0	0	3
1	0	0	2	1	0	0	2	1	0	2	1	0	6
0	1	0	2	0	1	0	4	0	1	2	0	1	6
1	1	0	6	1	1	0	2	1	1	6	1	1	3
0	0	1	2	0	0	1	6	0	0	2	0	0	3
1	0	1	6	1	0	1	8	1	0	2	1	0	6
0	1	1	6	0	1	1	4	0	1	2	0	1	6
1	1	1	4	1	1	1	2	1	1	6	1	1	3

- (a) Draw the factor graph corresponding to $p(x_2, x_3, x_4, x_5 | x_1 = 0, x_6 = 1)$ and give the tables defining the new factors $\phi_A^{x_1=0}(x_2, x_3)$ and $\phi_D^{x_6=1}(x_4)$ that you obtain.
- (b) Find $p(x_2 | x_1 = 0, x_6 = 1)$ using the elimination ordering (x_4, x_5, x_3) :
 - (i) Draw the graph for $p(x_2, x_3, x_5 | x_1 = 0, x_6 = 1)$ by marginalising x_4
Compute the table for the new factor $\tilde{\phi}_4(x_2, x_3, x_5)$
 - (ii) Draw the graph for $p(x_2, x_3 | x_1 = 0, x_6 = 1)$ by marginalising x_5
Compute the table for the new factor $\tilde{\phi}_{45}(x_2, x_3)$
 - (iii) Draw the graph for $p(x_2 | x_1 = 0, x_6 = 1)$ by marginalising x_3
Compute the table for the new factor $\tilde{\phi}_{453}(x_2)$
- (c) Now determine $p(x_2 | x_1 = 0, x_6 = 1)$ with the elimination ordering (x_5, x_4, x_3) :

- (i) Draw the graph for $p(x_2, x_3, x_4 \mid x_1 = 0, x_6 = 1)$ by marginalising x_5
 Compute the table for the new factor $\tilde{\phi}_5(x_4)$
- (ii) Draw the graph for $p(x_2, x_3 \mid x_1 = 0, x_6 = 1)$ by marginalising x_4
 Compute the table for the new factor $\tilde{\phi}_{54}(x_2, x_3)$
- (iii) Draw the graph for $p(x_2 \mid x_1 = 0, x_6 = 1)$ by marginalising x_3
 Compute the table for the new factor $\tilde{\phi}_{543}(x_2)$
- (d) Which variable ordering, (x_4, x_5, x_3) or (x_5, x_4, x_3) do you prefer?