These are exercises for self-study and exam preparation. All material is examinable unless otherwise mentioned.

Exercise 1. *Counting number of parameters*

Consider a pmf $p(a, z, q, h, e)$ that factorises as

$$p(a, z, q, h, e) = p(a)p(z)p(q|a, z)p(h|z)p(e|q)$$

We would like to count how many numbers we need to specify to represent $p(a, z, q, h, e)$ as a table when:

(a) *all variables are binary*

Solution. We need to specify all (conditional) pmfs factorisation. The (conditional) pmfs can be represented as tables for which we need the following number of parameters:

- 1 for $p(a)$, e.g. $p(a = 0)$ needs to be specified, while $p(a = 1) = 1 - p(a = 0)$ because of the normalisation constraint.
- 1 for $p(z)$
- 4 for $p(q|a, z)$, i.e. one for each of the four possible configurations of (a, z)
- 2 for $p(h|z)$, i.e. one for each of the two values of z
- 2 for $p(e|q)$

In total, there are 10 numbers to specify. This is in contrast to $2^5 - 1 = 31$ for a distribution without independencies.

(b) *variable a can take on 3 different values, variable h 5 different values, and all other variables are binary*

Solution. We count:

- 2 for $p(a)$, e.g. $p(a = 0)$ and $p(a = 1)$ need to be specified, while $p(a = 2) = 1 - p(a = 0) - p(a = 1)$ because of the normalisation constraint.
- 1 for $p(z)$
- 6 for $p(q|a, z)$, i.e. one for each of the six possible configurations of (a, z)
- 8 for $p(h|z)$, i.e. 5-1=4 parameters for each of the two values of z
- 2 for $p(e|q)$, i.e. one for each of the two values of q

In total, there are 19 numbers to specify. This is in contrast to $2^5 - 1 = 31$ for a distribution without independencies.