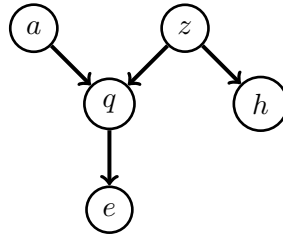


Exercise 1. Ordered and local Markov properties, d-separation

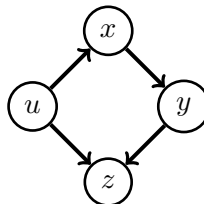
Consider the graph below:



- The ordering (z, h, a, q, e) is topological to the graph. What are the independences that follow from the ordered Markov property?
- What are the independences that follow from the local Markov property?
- The independence relations obtained via the ordered and local Markov property include $q \perp\!\!\!\perp h \mid \{a, z\}$. Verify the independence using d-separation.
- Use d-separation to check whether $a \perp\!\!\!\perp h \mid e$ holds.

Exercise 2. Flipping arrows

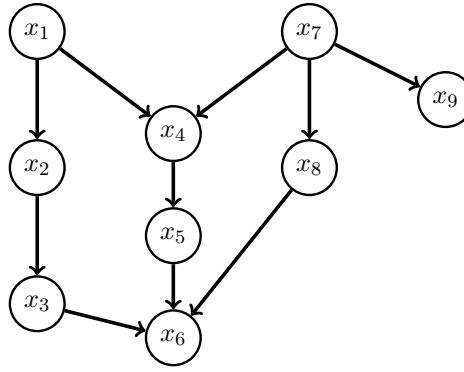
Consider the following graph:



- How does the corresponding directed graphical model factorise?
- List all independences encoded by the graph.
- How do the independences change when we flip the arrow from x to y so that it points the other way around, i.e. $y \rightarrow x$?

Exercise 3. Independences in directed graphical models

Consider the following directed acyclic graph.



For each of the statements below, determine whether it holds for all probabilistic models that factorise over the graph. Provide a justification for your answer.

- (a) $p(x_7|x_2) = p(x_7)$
- (b) $x_1 \not\perp x_3$
- (c) $p(x_1, x_2, x_4) \propto \phi_1(x_1, x_2)\phi_2(x_1, x_4)$ for some non-negative functions ϕ_1 and ϕ_2 .
- (d) $x_2 \perp x_9 \mid \{x_6, x_8\}$
- (e) $x_8 \perp \{x_2, x_9\} \mid \{x_3, x_5, x_6, x_7\}$
- (f) $\mathbb{E}[x_2 \cdot x_3 \cdot x_4 \cdot x_5 \cdot x_8 \mid x_7] = 0$ if $\mathbb{E}[x_8 \mid x_7] = 0$