

Exercise 1. Factorisation and independencies for undirected graphical models

Consider the undirected graphical model defined by the graph in Figure 1.

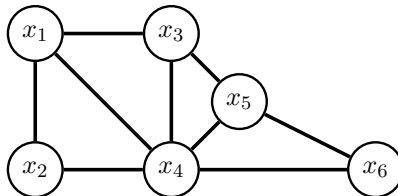


Figure 1: Graph for Exercise 1

- What is the set of Gibbs distributions that is induced by the graph?
- Let p be a pdf that factorises according to the graph. Does $p(x_3|x_2, x_4) = p(x_3|x_4)$ hold?
- Explain why $x_2 \perp\!\!\!\perp x_5 \mid x_1, x_3, x_4, x_6$ holds for all distributions that factorise over the graph.
- Assume you would like to approximate $\mathbb{E}(x_1 x_2 x_5 \mid x_3, x_4)$, i.e. the expected value of the product of x_1 , x_2 , and x_5 given x_3 and x_4 , with a sample average. Do you need to have joint observations for all five variables $x_1, \dots, x_5$?

Exercise 2. Factorisation from the Markov blankets

Assume you know the following Markov blankets for all variables $x_1, \dots, x_4, y_1, \dots, y_4$ of a pdf or pmf $p(x_1, \dots, x_4, y_1, \dots, y_4)$.

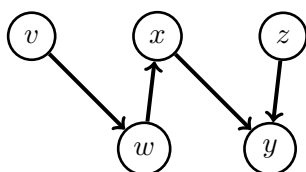
$$\text{MB}(x_1) = \{x_2, y_1\} \quad \text{MB}(x_2) = \{x_1, x_3, y_2\} \quad \text{MB}(x_3) = \{x_2, x_4, y_3\} \quad \text{MB}(x_4) = \{x_3, y_4\} \quad (1)$$

$$\text{MB}(y_1) = \{x_1\} \quad \text{MB}(y_2) = \{x_2\} \quad \text{MB}(y_3) = \{x_3\} \quad \text{MB}(y_4) = \{x_4\} \quad (2)$$

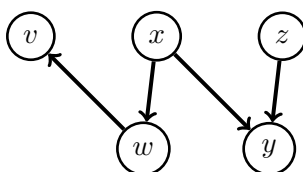
Assuming that p is positive for all possible values of its variables, how does p factorise?

Exercise 3. I-equivalence

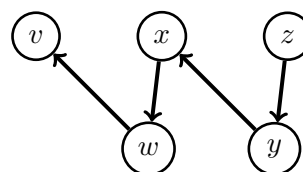
- Which of three graphs represent the same set of independencies? Explain.



Graph 1

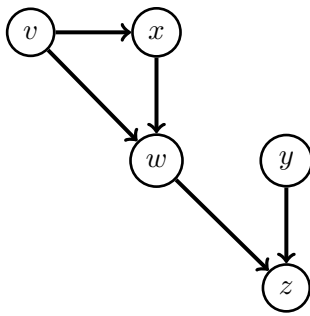


Graph 2

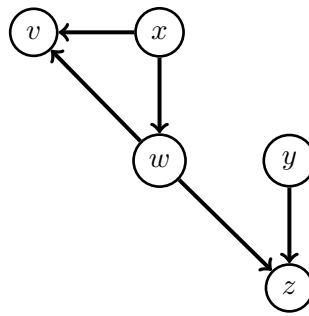


Graph 3

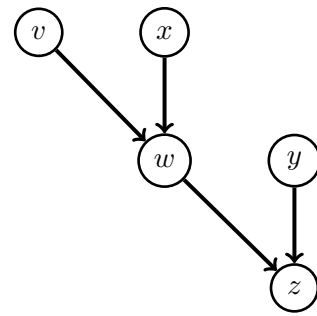
(b) Which of three graphs represent the same set of independencies? Explain.



Graph 1

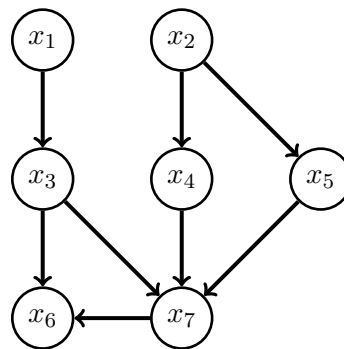


Graph 2



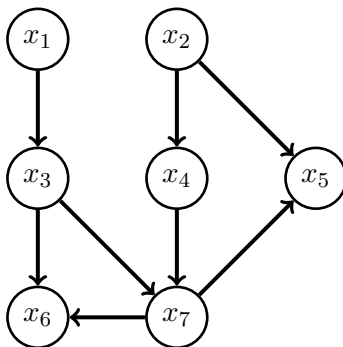
Graph 3

(c) Assume the graph below is a perfect map for a set of independencies $\mathcal{U}$.

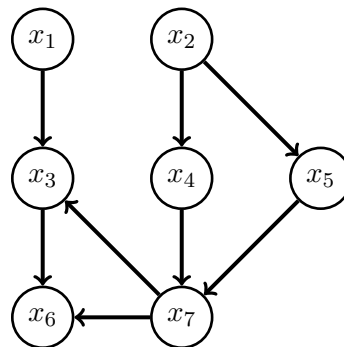


Graph 0

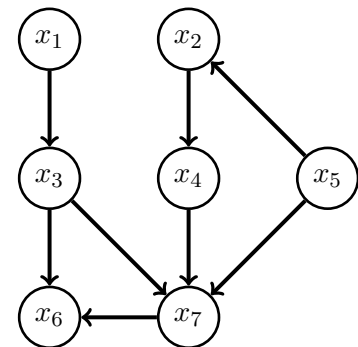
For each of the three graphs below, explain whether the graph is a perfect map, an I-map, or not an I-map for $\mathcal{U}$.



Graph 1



Graph 2



Graph 3