

Exercise 1. *Inference for the binary symmetric channel*

Let x_0 be a binary random variable taking on states 0 and 1 with probability $1/2$. A binary symmetric channel (BSC) flips an input bit with probability f , and leaves it unflipped with probability $1 - f$. Let x_1 be the result of passing x_0 through the BSC. Hence we have that $p(x_1 = 0|x_0 = 0) = p(x_1 = 1|x_0 = 1) = 1 - f$. Now suppose that x_1 is passed through another BSC (also with flip probability f) to yield x_2 . The graphical model is thus $x_0 \rightarrow x_1 \rightarrow x_2$.

- (a) You observe that $x_2 = 1$. Compute $p(x_0 = 1|x_2 = 1)$.