

Probabilistic Modelling and Reasoning

— Course Recap —

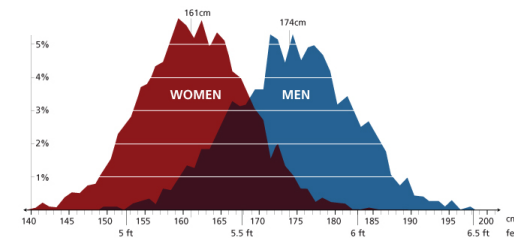
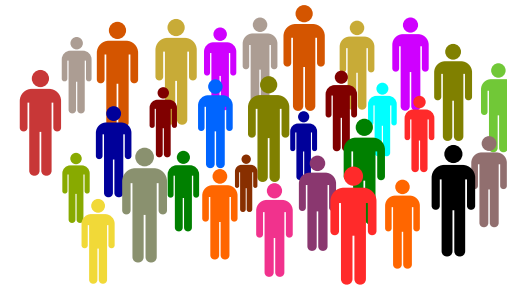
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Course recap

- ▶ We started the course with the basic observation that variability is part of nature.
- ▶ Variability leads to uncertainty when analysing or drawing conclusions from data.
- ▶ This motivates taking a probabilistic approach to modelling and reasoning.



Course recap

- ▶ Probabilistic modelling:
 - ▶ Identify the quantities that relate to the aspects of reality that you wish to capture with your model.
 - ▶ Consider them to be random variables, e.g. $\mathbf{x}, \mathbf{y}, \mathbf{z}$, with a joint pdf (pmf) $p(\mathbf{x}, \mathbf{y}, \mathbf{z})$.
- ▶ Probabilistic reasoning:
 - ▶ Assume you know that $\mathbf{y} \in \mathcal{E}$ (measurement, evidence)
 - ▶ Probabilistic reasoning about \mathbf{x} then consists in computing

$$p(\mathbf{x}|\mathbf{y} \in \mathcal{E})$$

or related quantities like its maximiser or posterior expectations.

Course recap

- ▶ Principled framework but direct implementation quickly runs into computational issues.

- ▶ For example,

$$p(\mathbf{x}|\mathbf{y}_o) = \frac{\sum_{\mathbf{z}} p(\mathbf{x}, \mathbf{y}_o, \mathbf{z})}{\sum_{\mathbf{x}, \mathbf{z}} p(\mathbf{x}, \mathbf{y}_o, \mathbf{z})}$$

cannot be computed if $\mathbf{x}, \mathbf{y}, \mathbf{z}$ each are $d = 500$ dimensional, and if each element of the vectors can take $K = 10$ values.

- ▶ The course had four main topics.

Topic 1: Representation We discussed reasonably weak assumptions to efficiently represent $p(\mathbf{x}, \mathbf{y}, \mathbf{z})$.

- ▶ Two classes of assumptions: independence and parametric assumptions.
- ▶ Directed and undirected graphical models
- ▶ Independencies encoded by the graphs. Their expressive power.
- ▶ Causality and graphical models
- ▶ Factor graphs

Course recap

Topic 2: Exact inference We have seen that the independence assumptions allow us, under certain conditions, to efficiently compute the posterior probability or derived quantities.

- ▶ Variable elimination for general factor graphs
- ▶ Inference when the model can be represented as a factor tree (message passing algorithms)
- ▶ Application to Hidden Markov models (alpha and the alpha-beta recursion)

Topic 3: Learning We discussed methods to learn probabilistic models from data by introducing parameters and learning them from data.

- ▶ Learning by Bayesian inference
- ▶ Learning by parameter estimation
- ▶ Likelihood function
- ▶ Factor analysis and independent component analysis

Topic 4: Approximate inference and learning Intractable integrals may hinder inference and likelihood-based learning.

- ▶ Intractable integrals may be due to unobserved variables or intractable partition functions.
- ▶ Variational approaches to learning and inference
- ▶ EM algorithm and its application to hidden Markov models
- ▶ Learning of deep latent variable models and variational autoencoders