These notes are intended to give a summary of relevant concepts from the lectures which are helpful to complete the exercises. It is not intended to cover the lectures thoroughly. Learning this content is not a replacement for working through the lecture material and the exercises.

Structural Causal Model: A structural causal model (SCM) M is given by the set of assignments

$$X_i \coloneqq f_i(Pa_i, U_i),$$

one for each variable in the network. Here the f_is are deterministic functions, and the U_is are jointly independent noise variables.

Interventions and the do-operator For a given node X in M the assignment $X \coloneqq f(Pa, U)$ is replaced by by $X \coloneqq x$. We denote this modified model as $M' = M[X \coloneqq x]$ or $M' = M; do(X \coloneqq x)$. Graphically, the operation eliminates all incoming edges into X.

After applying the do-operator, we obtain probabilities for an event E under the intervention as $p_{M[X:=x]}(E)$, which can also be written as $p(E|do(X \coloneqq x))$.

Counterfactuals: General recipe

1. **Abduction:** Condition the joint distribution of the exogenous variables $U = (U_1, \ldots, U_d)$ on the event $E = e$ to obtain $p(U|E = e)$.
2. **Action:** Perform the do-intervention $X \coloneqq x$ in M resulting in the model $M' = M[X \coloneqq x]$ and the modified graph.
3. **Prediction:** Compute the target counterfactual using the noise distribution $p(U|E = e)$ in M'.

I–map — The set of independencies that a graph K asserts is denoted $\mathcal{I}(K)$. K is said to be an independency map (I–map) for a set of independencies $\mathcal{U}$ if,

$$\mathcal{I}(K) \subseteq \mathcal{U} \tag{1}$$

A complete graph is an I–map since it makes no assertions, this means that an I–map is not necessarily useful.

While the set of "target" independencies $\mathcal{U}$ can be specified in any way, they are often the independencies that a certain distribution p satisfies. This set of independencies is denoted by $\mathcal{I}(p)$.

Minimal I–map — A "sparsified" I-map: A graph K such that if any edge is removed, $\mathcal{I}(K) \not\subseteq \mathcal{U}$.

P–map — K is said to be a perfect map (P–map) for a set of independencies $\mathcal{U}$ if $\mathcal{I}(K) = \mathcal{U}$

Constructing minimal I–maps

Undirected graphs — $\forall x_i \in N$, determine $\mathrm{MB}(x_i)$ and connect x_i to all variables in $\mathrm{MB}(x_i)$.

Directed graphs — Assume an ordering $\mathbf{x} = (x_1, \ldots, x_d)$, then $\forall x_i \in \mathbf{x}$ set pa_i to π_i, where π_i is a minimal subset of the pre_i such that

$$x_i \perp\!\!\!\perp \{\mathrm{pre}_i \setminus \pi_i\} \mid \pi_i \tag{2}$$

I–equivalence

Undirected graphs — $\mathcal{I}(H_1)$ and $\mathcal{I}(H_2)$ are I–equivalent *iff* they have the same skeleton.

Directed graphs — $\mathcal{I}(G_1)$ and $\mathcal{I}(G_2)$ are I–equivalent *iff* they have the same skeleton and set of immoralities.

Undirected and directed graphs — $\mathcal{I}(H)$ and $\mathcal{I}(G)$ are I–equivalent *iff* they have the same skeleton and the DAG G does not have immoralities.

- Skeleton – graph without arrow heads, i.e. connections irrespective of direction
- Immoralities – the set of collider nodes without covering edge (without “married parents”)