These notes summarise selected lecture concepts and are not a substitute for working through the lecture slides, tutorials, and self-study exercises. Feel free to personalise and develop them into your own summary sheet.

Causal DAGs — Causal DAGs are DAGs where the arrows are assumed to represent a causal direction.

Graph surgery — Intervening on variable x_k removes all incoming arrows into the node and makes it a root variable.

Postinterventional distribution — Assume we intervene on x_k by setting $x_k \sim p'(x_k)$ for intervention distribution (randomisation scheme) p'. The resulting distribution is
$p(\mathbf{x}; do(x_k) \sim p') = \prod_{i \neq k} p(x_i|\text{pa}_i) \cdot p'(x_k)$

Atomic interventions — Atomic interventions correspond to using a point mass for $p'(x_k)$, setting x_k to a fixed value, e.g. a. Notation for the postinterventional distr: $p(\mathbf{x}; do(x_k) = a)$

Inverse probability weighting — A method to turn observational data $\mathbf{x}^{(i)} \sim p(\mathbf{x})$ into samples from $p(\mathbf{x}; do(x_k) \sim p')$ by reweighting each $\mathbf{x}^{(i)}$ with $w^{(i)} = p'(x_k^{(i)})/p(x_k^{(i)}|\text{pa}_k^{(i)})$.

Adjustment for direct causes — A method to compute how intervening on x_k changes the distribution of x_i:

$$p(x_i; do(x_k) \sim p') = \mathbb{E}_{p(\text{pa}_k)p'(x_k)}\left[p(x_i|x_k, \text{pa}_k)\right] \qquad p(x_i; do(x_k) = a) = \mathbb{E}_{p(\text{pa}_k)}\left[p(x_i|x_k = a, \text{pa}_k)\right]$$

Backdoor paths — A backdoor path from x to y is any (undirected) path that begins with an arrow into x, typically transmitting spurious association via common causes rather than the causal effect of x on y. Open backdoor paths generally make the conditional and postinterventional distribution different.

Action/observation exchange — Let $G_{\underline{x_k}}$ be the graph where all outgoing arrows from x_k are removed. If $x_i \perp\!\!\!\perp x_k|\mathbf{z}$ in $G_{\underline{x_k}}$ then $p(x_i|\mathbf{z}; do(x_k) = a) = p(y|\mathbf{z}, x_k = a)$

Backdoor adjustment — This is a method that generalises the adjustment for directed causes. If $\mathbf{z}$ satisfies (1) $x_i \perp\!\!\!\perp x_k|\mathbf{z}$ in $G_{\underline{x_k}}$ (2) no component of $\mathbf{z}$ is a descendant of x_k, then $p(x_i; do(x_k) = a) = \mathbb{E}_{p(\mathbf{z})}\left[p(x_i|x_k = a, \mathbf{z})\right]$.