These notes summarise selected lecture concepts and are not a substitute for working through the lecture slides, tutorials, and self-study exercises. Feel free to personalise and develop them into your own summary sheet.

I–map — The set of independencies that a graph K asserts is denoted $\mathcal{I}(K)$. K is said to be an independency map (I–map) for a set of independencies $\mathcal{U}$ if,

$$\mathcal{I}(K) \subseteq \mathcal{U} \tag{1}$$

A complete graph is an I–map since it makes no assertions, this means that an I–map is not necessarily useful.

While the set of "target" independencies $\mathcal{U}$ can be specified in any way, they are often the independencies that a certain distribution p satisfies. This set of independencies is denoted by $\mathcal{I}(p)$.

Minimal I–map — A "sparsified" I-map: A graph K such that if any edge is removed, $\mathcal{I}(K) \nsubseteq \mathcal{U}$.

P–map — K is said to be a perfect map (P–map) for a set of independencies $\mathcal{U}$ if $\mathcal{I}(K) = \mathcal{U}$

Constructing minimal I–maps

Undirected graphs — $\forall x_i \in N$, determine $\text{MB}(x_i)$ and connect x_i to all variables in $\text{MB}(x_i)$.

Directed graphs — Assume an ordering $\mathbf{x} = (x_1, \ldots, x_d)$, then $\forall x_i \in \mathbf{x}$ set pa_i to π_i, where π_i is a minimal subset of the pre_i such that

$$x_i \perp\!\!\!\perp \{\text{pre}_i \setminus \pi_i\} \mid \pi_i \tag{2}$$

I–equivalence

Undirected graphs — $\mathcal{I}(H_1)$ and $\mathcal{I}(H_2)$ are I–equivalent *iff* they have the same skeleton.

Directed graphs — $\mathcal{I}(G_1)$ and $\mathcal{I}(G_2)$ are I–equivalent *iff* they have the same skeleton and set of immoralities.

Undirected and directed graphs — $\mathcal{I}(H)$ and $\mathcal{I}(G)$ are I–equivalent *iff* they have the same skeleton and the DAG G does not have immoralities.

- Skeleton – graph without arrow heads, i.e. connections irrespective of direction
- Immoralities – the set of collider nodes without covering edge (without "married parents")