Probabilistic Modelling and Reasoning Notes (HMM Inference)

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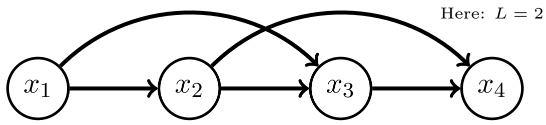
These notes summarise selected lecture concepts and are not a substitute for working through the lecture slides, tutorials, and self-study exercises. Feel free to personalise and develop them into your own summary sheet.

Markov chains — A distribution factorised such that each variable x_i depends on L previous (contiguous) nodes $\{x_{i-L}, \ldots, x_{i-1}\}$

$$p(x_1, \ldots, x_d) = \prod_{i=1}^{d} p(x_i \mid x_{i-L}, \ldots, x_{i-1})$$

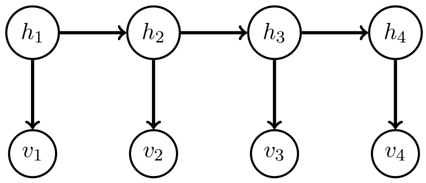


For $L = 1$ we have a 1st-order Markov chain, $p(x_1, \ldots, x_d) = \prod_{i=1}^{d} p(x_i \mid x_{i-1})$

The transition distribution $p(x_i, \mid x_{i-1})$ gives the probability of transitioning to different states. However, if this does not depend on i, then the Markov chain is said to be homogeneous.

Hidden Markov model (HMM) — A 1st-order Markov chain on latent variables h_i (hiddens), with an additional set of visible variables v_i that represent observations. An emission distribution $p(v_i \mid h_i)$ gives the probabilities of the observations v_i (visibles) taking different values, if the observations are real-valued then $p(v_i \mid h_i)$ will be a probability density function.

$$p(h_{1:d}, v_{1:d}) = p(v_1 \mid h_1) p(h_1) \prod_{i=2}^{d} p(v_i \mid h_i) p(h_i \mid h_{i-1})$$



An HMM is said to be stationary if its transition and emission distributions don't depend on i.

Alpha-recursion A recursive process that propagates information forwards, from h_{s-1} to h_s

$$\alpha(h_s) = p(v_s \mid h_s) \sum_{h_{s-1}} p(h_s \mid h_{s-1}) \alpha(h_{s-1}) = p(h_s, v_{1:s}) \propto p(h_s | v_{1:s}) \tag{1}$$

$$\alpha(h_1) = p(h_1) p(v_1 \mid h_1) = p(h_1, v_1) \propto p(h_1 \mid v_1) \tag{2}$$

Used to compute $p(h_t | v_{1:t})$ (filtering).

Beta-recursion A recursive process that propagates information backwards, from h_s to h_{s-1}

$$\beta(h_{s-1}) = \sum_{h_s} p(v_s \mid h_s) p(h_s \mid h_{s-1}) \beta(h_s) = p(v_{s:u} | h_{s-1}) \tag{3}$$

$$\beta(h_u) = 1 \tag{4}$$

Together with the alpha-recursion, used to compute $p(h_t | v_{1:u})$ (smoothing).