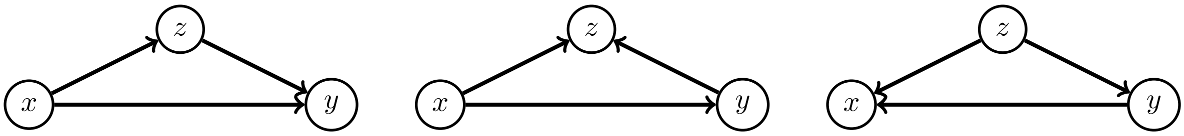
These are exercises for self-study and exam preparation. All material is examinable unless otherwise mentioned.

Exercise 1. ***Computing postinterventional distributions***

Consider the following causal DAGs for three discrete-valued random variables x, y, z:



(a) (b) (c)

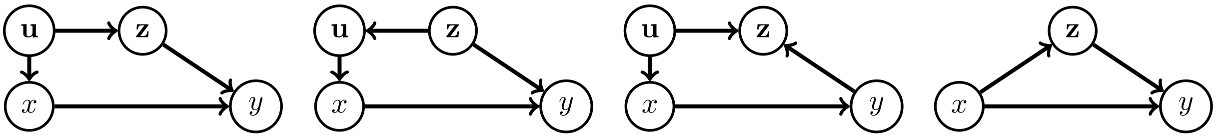
(a) Compute $p(y; do(x) = a)$ for DAG (a). Express the result in terms of the conditional probability distributions $p(x_i|\text{pa}_i)$ of the graphical model defined the DAG.

(b) Compute $p(y; do(x) = a)$ for DAG (b). Express the result in terms of the conditional probability distributions $p(x_i|\text{pa}_i)$ of the graphical model defined the DAG.

(c) Compute $p(y; do(x) = a)$ for DAG (c). Express the result in terms of the conditional probability distributions $p(x_i|\text{pa}_i)$ of the graphical model defined the DAG.

Exercise 2. ***Backdoor adjustment***

For each of the following DAGs G, explain whether $\mathbf{z}$ can be used to compute $p(y; do(x))$ via backdoor adjustment.



(a) (b) (c) (d)

Exercise 3. ***Backdoor adjustment for non-atomic interventional distributions***

The backdoor adjustment criterion says that if $\mathbf{z}$ satisfies

1. $x_i \perp\!\!\!\perp x_k|\mathbf{z}$ in $G_{\underline{x_k}}$, and
2. no component of $\mathbf{z}$ is a descendant of x_k,

then $p(x_i; do(x_k) = a) = \mathbb{E}_{p(\mathbf{z})}\left[p(x_i|x_k = a, \mathbf{z})\right]$. Here, $G_{\underline{x_k}}$ denotes the graph where all outgoing arrows from x_k are removed.

Extend this result to $p(x_i; do(x_k) \sim p'(x_k))$ where $p'(x_k)$ is a general interventional distribution. For simplicity, you can assume that the random variables are discrete-valued.