Probabilistic Modelling and Reasoning
— Course Recap —

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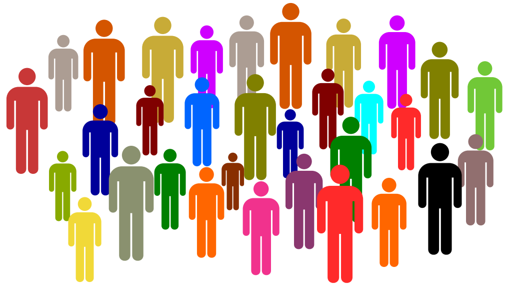
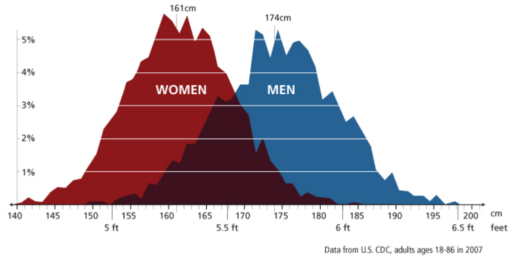
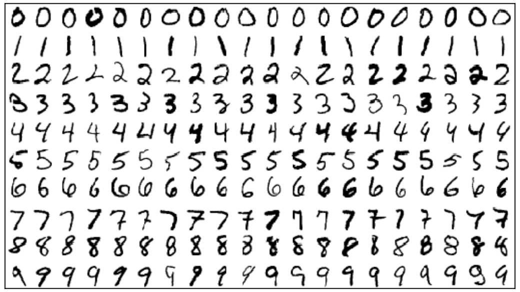
Course recap

- We started the course with the basic observation that variability is part of nature.
- Variability leads to uncertainty when analysing or drawing conclusions from data.
- This motivates taking a probabilistic approach to modelling and reasoning.







Course recap

- Probabilistic modelling:
  - Identify the quantities that relate to the aspects of reality that you wish to capture with your model.
  - Consider them to be random variables, e.g. $\mathbf{x}, \mathbf{y}, \mathbf{z}$, with a joint pdf (pmf) $p(\mathbf{x}, \mathbf{y}, \mathbf{z})$.
- Probabilistic reasoning:
  - Assume you know that $\mathbf{y} \in \mathcal{E}$ (measurement, evidence)
  - Probabilistic reasoning about $\mathbf{x}$ then consists in computing

    $$p(\mathbf{x}|\mathbf{y} \in \mathcal{E})$$

    or related quantities like its maximiser or posterior expectations.

Course recap

- Principled framework but direct implementation quickly runs into computational issues.
- For example,
$$p(\mathbf{x}|\mathbf{y}_o) = \frac{\sum_{\mathbf{z}} p(\mathbf{x}, \mathbf{y}_o, \mathbf{z})}{\sum_{\mathbf{x},\mathbf{z}} p(\mathbf{x}, \mathbf{y}_o, \mathbf{z})}$$
cannot be computed if $\mathbf{x}, \mathbf{y}, \mathbf{z}$ each are $d = 500$ dimensional, and if each element of the vectors can take $K = 10$ values.
- The course had five main topics.
  1. Representation
  2. Exact inference
  3. Actions and decision making
  4. Learning
  5. Approximate inference and learning

Topic 1: Representation

We discussed reasonably weak assumptions to efficiently represent $p(\mathbf{x}, \mathbf{y}, \mathbf{z})$.

- Two classes of assumptions: independence and parametric assumptions.
- They are orthogonal to each other and can be combined.
- Directed and undirected graphical models
- Independencies encoded by the graphs. Their expressive power.
- Connection between independencies and the factorisation of the pdf/pmf.
- The chain rule and autoregressive models.

Topic 2: Exact inference

We have seen that the independence assumptions allow us, under certain conditions, to efficiently compute conditional /posterior distributions or derived quantities.

- Factor graphs to represent factorisations
- Variable elimination for general factor graphs
- Inference when the model can be represented as a factor tree (message passing algorithms)
- Application to Hidden Markov models

Topic 3: Actions and decision making

We then discussed how to turn (conditional) distributions into actions.

- We modelled actions as interventions in directed graphical models and used decision theory to derive optimal actions.
- Causal DAGs and how interventions change their structure
- Methods to compute the effect of interventions, i.e. the postinterventional distribution.
- Turning beliefs into actions by minimising the expected loss.
- Standard loss functions and the corresponding optimal actions.

Topic 4: Learning

Inference and decision making requires probabilistic models. We discussed methods to learn them from data.

- Learning by Bayesian inference
- Learning by parameter estimation
- Likelihood function
- Factor analysis and independent component analysis

Topic 5: Approximate inference and learning

We discussed computational intractabilities in inference and learning: intractable integrals often preclude exact inference and likelihood-based learning.

- Intractable integrals may be due to unobserved variables or intractable partition functions.
- Variational approaches to learning and inference, including mean-field VI and the EM algorithm
- Application of the EM algorithm to Hidden Markov Models
- Learning of deep latent variable models and variational autoencoders
- Monte Carlo integration and sampling