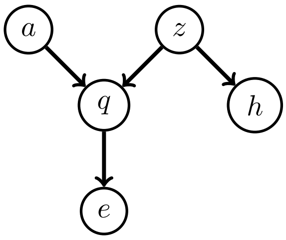
Exercise 1. ***Ordered and local Markov properties, d-separation***

Consider the graph below:



(a) The ordering (z, h, a, q, e) is topological to the graph. What are the independencies that follow from the ordered Markov property?

(b) What are the independencies that follow from the local Markov property?

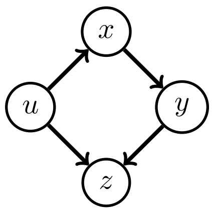
(c) The independency relations obtained via the ordered and local Markov property include $q \perp\!\!\!\perp h \mid \{a, z\}$. Verify the independency using d-separation.

(d) Use d-separation to check whether $a \perp\!\!\!\perp h \mid e$ holds.

(e) Assume all variables in the graph are binary. How many numbers do you need to specify, or learn from data, in order to fully specify the probability distribution?

Exercise 2. ***Flipping arrows***

Consider the following graph:



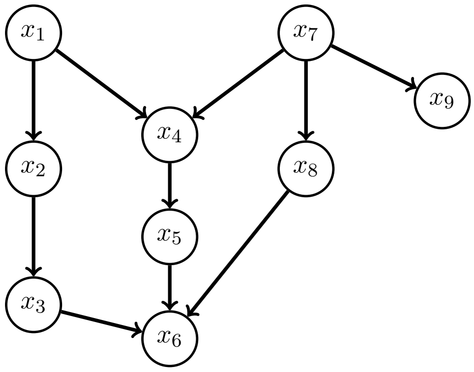
(a) How does the corresponding directed graphical model factorise?

(b) List all independencies encoded by the graph.

(c) How do the independencies change when we flip the arrow from x to y so that it points the other way around, i.e. $y \to x$.?

Exercise 3. ***Independencies in directed graphical models***

Consider the following directed acyclic graph.



For each of the statements below, determine whether it holds for all probabilistic models that factorise over the graph. Provide a justification for your answer.

(a) $p(x_7|x_2) = p(x_7)$

(b) $x_1 \not\perp\!\!\!\perp x_3$

(c) $p(x_1, x_2, x_4) \propto \phi_1(x_1, x_2)\phi_2(x_1, x_4)$ for some non-negative functions ϕ_1 and ϕ_2.

(d) $x_2 \perp\!\!\!\perp x_9 \mid \{x_6, x_8\}$

(e) $x_8 \perp\!\!\!\perp \{x_2, x_9\} \mid \{x_3, x_5, x_6, x_7\}$

(f) $\mathbb{E}[x_2 \cdot x_3 \cdot x_4 \cdot x_5 \cdot x_8 \mid x_7] = 0$ if $\mathbb{E}[x_8 \mid x_7] = 0$