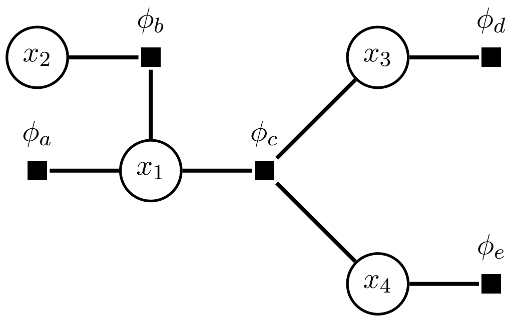
Exercise 1. ***Sum-product message passing***

The following factor graph represents a Gibbs distribution over four binary variables $x_i \in \{0, 1\}$.



The factors ϕ_a, ϕ_b, ϕ_d are defined as follows:

x_1	ϕ_a
0	2
1	1

x_1	x_2	ϕ_b
0	0	5
1	0	2
0	1	2
1	1	6

x_3	ϕ_d
0	1
1	2

and $\phi_c(x_1, x_3, x_4) = 1$ if $x_1 = x_3 = x_4$, and is zero otherwise.

For all questions below, justify your answer:

(a) Compute the values of $\mu_{x_2 \to \phi_b}(x_2)$ for $x_2 = 0$ and $x_2 = 1$.

(b) Assume the message $\mu_{x_4 \to \phi_c}(x_4)$ equals

$$\mu_{x_4 \to \phi_c}(x_4) = \begin{cases} 1 & \text{if } x_4 = 0 \\ 3 & \text{if } x_4 = 1 \end{cases}$$

Compute the values of $\phi_e(x_4)$ for $x_4 = 0$ and $x_4 = 1$.

(c) Compute the values of $\mu_{\phi_c \to x_1}(x_1)$ for $x_1 = 0$ and $x_1 = 1$.

(d) The message $\mu_{\phi_b \to x_1}(x_1)$ equals

$$\mu_{\phi_b \to x_1}(x_1) = \begin{cases} 7 & \text{if } x_1 = 0 \\ 8 & \text{if } x_1 = 1 \end{cases}$$

What is the probability that $x_1 = 1$, i.e. $p(x_1 = 1)$?

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Exercise 2. ***Viterbi algorithm***

For the hidden Markov model

$$p(h_{1:t}, v_{1:t}) = p(v_1|h_1)p(h_1)\prod_{i=2}^{t} p(v_i|h_i)p(h_i|h_{i-1})$$

assume you have observations for v_i, $i = 1, \ldots, t$. Use the max-sum algorithm to derive an iterative algorithm to compute

$$\hat{\mathbf{h}} = \underset{h_1,\ldots,h_t}{\operatorname{argmax}}\, p(h_{1:t}|v_{1:t}) \tag{1}$$

Assume that the latent variables h_i can take K different values, e.g. $h_i \in \{0, \ldots, K-1\}$. The resulting algorithm is known as Viterbi algorithm.

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